CTB3350

Open Channel Flow

Tentamenbundel Civiele Techniek
Het Gezelschap "Practische Studie"

LET OP! EEN REPRODUCERENDE LEERSTIJL IS SCHADELIJK VOOR DE ACADEMISCHE VORMING

Juni 2019
April 2019
April 2018
Juni 2017
April 2017
April 2016
Juni 2015
April 2015
April 2014

Vul dit schema in om een overzicht te hebben van welke onderwerpen van het vak al goed gaan en welke je nog moet oefenen

	Juni 2017		April 2017		April 2016		Juni 2015		April 2015		April 2014	
Vraag	Onderwerp	Goed?	Onderwerp	Goed?	Onderwerp	Goed?	Onderwerp	Goed?	Onderwerp	Goed?	Onderwerp	Goed?
1												
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	Juni 2019		April 2019		April 2018	
Vraag	Onderwerp	Goed?	Onderwerp	Goed?	Onderwerp	Goed?
1						
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Question I - (tsunami) *use a new answer form*

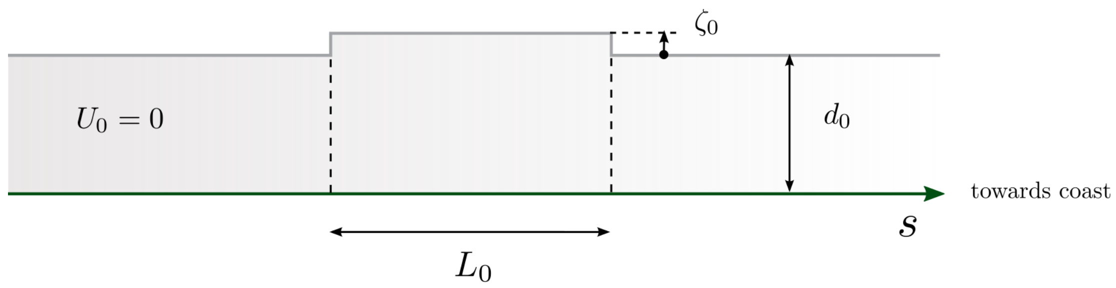
Following a sub sea earthquake, the water level in an ocean increases locally and instantaneously with several meters. The area where the initial disturbance of the surface level is noticeable has a horizontal extent L_0 of 200 km. The average water depth d_0 in the corresponding region amounts to 1500 m.

I.1$^{1.0}$ Explain - using a small calculation - that the tsunami resulting from this earthquake is a *long* wave.

The propagation of the tsunami is governed by the shallow water equations. In the given case, several simplifications of these equations are possible.

I.2$^{1.0}$ Mention the three most important simplifications that apply in the given situation and give an explanation why they are valid.

The initial surface elevation is approximately uniform along the (straight) coastline. The remainder of this question will therefore consider the situation per unit width with the waves propagating in the direction perpendicular to the coastline. The initial surface elevation ζ_0 amounts to 3.0 m in the region of the earthquake and is zero elsewhere, while the initial flow velocity U_0 is zero everywhere. See the accompanying figure for a sketch of the initial condition at time $t = 0$, just after the earthquake.



I.3$^{1.5}$ Compute and plot the surface elevation $\zeta(s)$ and the corresponding depth averaged flow velocity $U(s)$ at time $t = 600$ s.

Towards the coast, the water depth decreases gradually. According to Green's law the surface elevation ζ and length L of the wave component travelling towards the coast change, such that the quantities $\rho g L \zeta^2$ and L/c remain constant when following this wave.

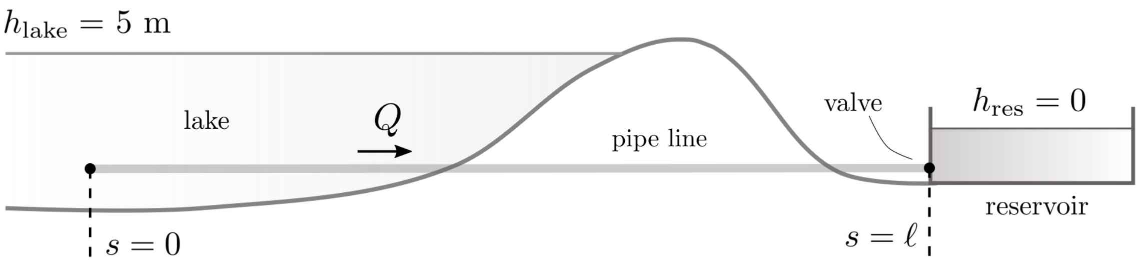
I.4$^{1.0}$ Give a physical interpretation of the formulated quantities, and explain the forthcoming principle that is used to derive Green's law.

In the direction of the coast, the water depth decreases gradually to a depth d_1 of 15 m.

I.5$^{1.5}$ Use Green's law in combination with linear wave theory to compute the surface level elevation and flow velocity of the tsunami at this depth, and discuss the validity of the result by reconsidering the topics addressed in Question I.2.

Question II - (fresh water intake) *use a new answer form*

Raw drinking water (bulk modulus 2.2 GPa) is extracted from a fresh water lake via a closed concrete pipe system feeding a lower laying storage reservoir. The pipe has a length ℓ and the streamwise coordinate s is defined positive in inflow direction. At the outflow end ($s = \ell$) an emergency valve is situated that closes automatically if the quality of the lake water becomes unacceptable. See the accompanying figure.



The concrete pipe has a uniform cross section and is infinitely rigid. Experience with similar intake pipes has shown that too abrupt changes of the discharge can cause structural failure of the pipe.

II.1$^{1.0}$ Compute the speed of pressure waves in an infinitely rigid pipe, and explain why such pipes are particularly prone to structural failure when the discharge is varied too abruptly.

Wall-friction losses in the pipe can be neglected. At the intake ($s = 0$), the energy loss during inflow is zero while it equals the local velocity head during outflow. The discharge relation of the valve leads to $h = h_{\text{res}} + \xi|U|U/2g$ at $s = \ell$, where ξ is the head loss coefficient which initially equals 9.

II.2$^{1.0}$ Compute the initial piezometric level h_0 and the flow velocity U_0 in the pipe using the above boundary conditions and simplifications.

To study the pressure waves in the pipe that result from an emergency closure of the valve, the method of characteristics is used. A closure procedure is considered where at time $t = 0$ the valve is closed fully and abruptly while the boundary condition at $s = 0$ remains unchanged.

II.3$^{2.0}$ Construct the corresponding s,t-plane and U,h-state diagram for the time interval $0 < t < 6t_\ell$, where t_ℓ is the travel time of pressure waves in the pipe line; the initial - and boundary conditions have to be plotted exactly, but the characteristic relations in the state diagram may be sketched schematically.

If the pressure in the pipe becomes too high the pipe wall may break. This can be avoided by closing the valve *partially*, which results in a discharge coefficient $\xi' < \infty$ of the valve for $t > 0$.

II.4$^{1.5}$ Indicate where and when the maximum piezometric level occurs after (partially) closing the valve, and explain why a partial closure of the valve will reduce the corresponding value of the maximum piezometric level; provide your answer with a clear sketch in the previous U,h-state diagram.

As a design criterion, the maximum piezometric level in the pipe may not exceed +40.0 m. This determines the maximum value of the discharge coefficient ξ' resulting from the partial closure.

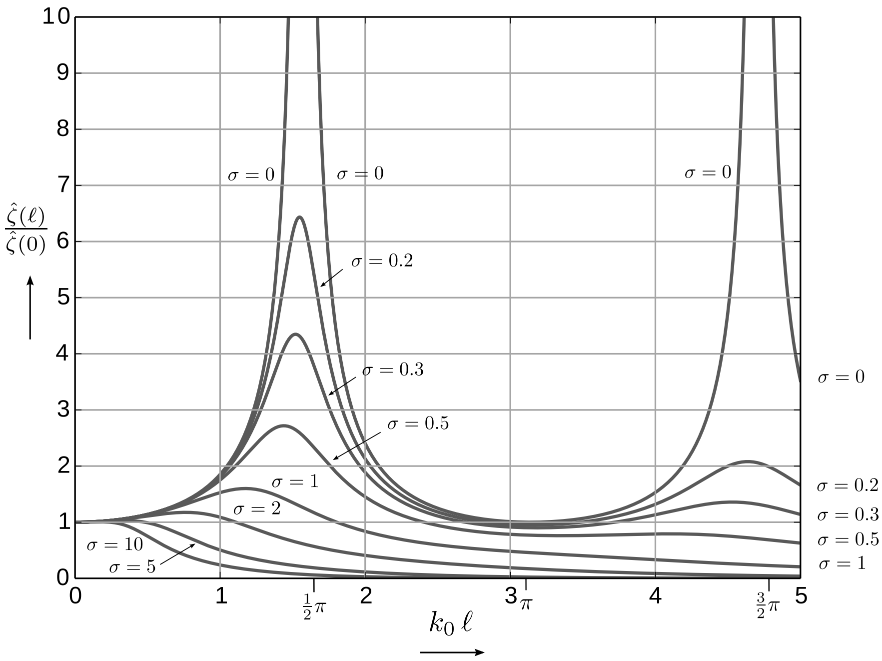
II.5$^{1.5}$ For the initial conditions of Question II.3, compute the maximum allowable discharge coefficient ξ' of the valve after partial closure for which the design criterion is satisfied.

Question III - (tidal river) *use a new answer form*

A tidal river (M_2) provides access to a large port. In order to improve the accessibility of the port for ships, the depth of the tidal river's main channel will be increased. This will have a number of consequences for the hydrodynamics of the tidal river which will be thoroughly investigated before the plans are executed.

III.1$^{1.0}$ Describe the two dimensionless parameters that quantify the principal features of tidal wave propagation in a tidal river, and explain how these parameters are likely affected if the main channel is deepened.

For a first assessment of the effects of the deepening on the tidal wave, a one-dimensional, linearized shallow water model is used. The tidal river - which is closed at one side - is schematized using a uniform cross section. The response factor for such a system is plotted in the figure below.



For the model of the tidal river a length ℓ of 65 km, a conveyance cross section A_c of 17,000 m^2 and a storage width B of 2,000 m are used. The surface level amplitude at sea amounts to 0.80 m and the observed surface level amplitude at the closed end equals 1.20 m.

III.2$^{1.5}$ Use the graph of the response factor - supplemented with some calculations - to determine the value of the resistance factor κ to be used in the model in order to reproduce the correct surface level amplitude at the closed end for the given amplitude at sea.

III.3$^{1.5}$ For the selected value of κ, compute the corresponding discharge amplitude $\hat{Q}$ at the mouth of the tidal river.

Deepening the main channel involves an increase of the conveyance cross section and the hydraulic radius, both with 15%. All other parameters are assumed to remain unchanged, except the resistance factor κ which can not be determined beforehand.

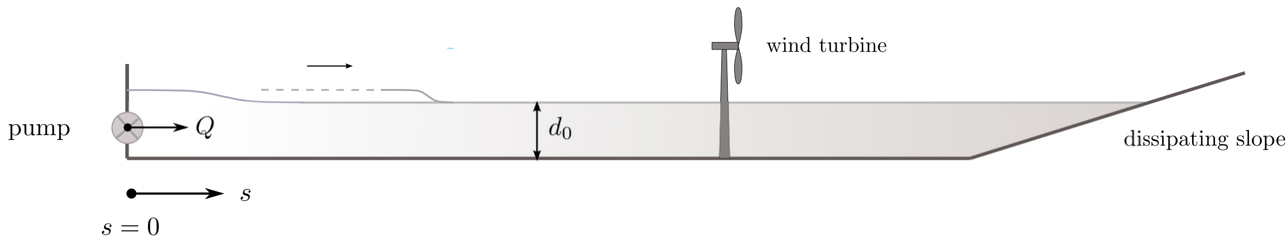
III.4$^{1.0}$ Explain the reason for this mathematical problem and describe the sequence of computational steps that have to be performed in order to determine the new value of κ.

After performing the steps referred to in Question III.4, it turns out that for the situation with the deepened channel the value of κ amounts to $1.15 \times 10^{-4}\,\mathrm{s}^{-1}$.

III.5$^{1.0}$ Compute how much the discharge amplitude at the mouth of the tidal river has changed due to the deepening works; note that it has been assumed that c_f remains unaffected by the deepening.

Question IV - (wave tests monopile) *use a new answer form*

A monopile (foundation) for an offshore wind turbine is tested in an experimental flume. In particular, the effect of waves on the construction is investigated. In a first series of experiments, a compression wave is generated in the flume which should break exactly at the location of the monopile in order to generate a maximum impact force. See the accompanying figure.



Starting from rest, the compression wave is generated by gradually increasing the discharge at the entrance of the flume. The compression wave steepens until it breaks.

IV.1$^{1.0}$ Explain the physical mechanism that causes steepening of the compression wave, and mention the terms of the shallow water equations that are responsible for this behaviour.

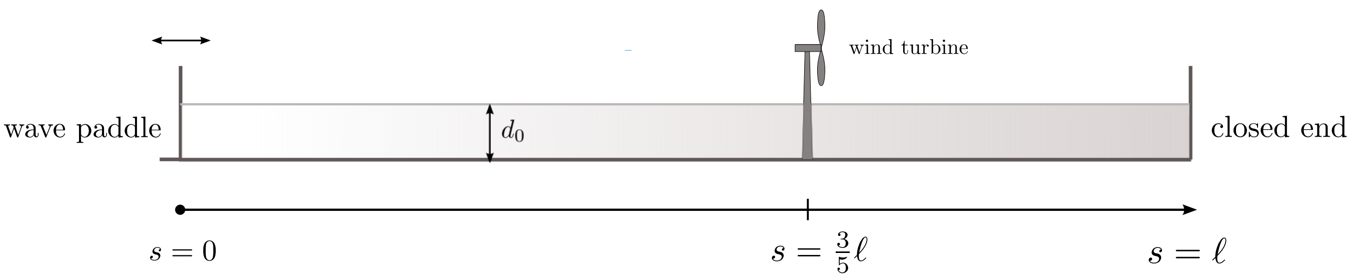
By varying the discharge Q in a specific way in time, the location where the compression wave breaks can be adjusted. The required discharge variation - to let the wave break at the monopile - can be determined by using the method of characteristics.

IV.2$^{1.0}$ Make a schematic, yet physically accurate plot of the positive characteristics corresponding to the compression wave issued by the varied discharge, and explain how the location and moment of wave breaking can be determined from this plot; provide your plot with some relevant comments.

After breaking (at the monopile), the wave continues as a bore with a propagation speed $c = \sqrt{g(d_1/d_0)(d_0+d_1)/2}$, where d_0 and d_1 are the depths before and after passage of the bore, respectively. The undisturbed depth d_0 of the flume amounts to 35 cm and the specific discharge q of the bore equals $0.20\,\mathrm{m^2/s}$.

IV.3$^{1.5}$ Compute the height $d_1 - d_0$ of the bore.

A second series of experiments concerns the impact of standing harmonic waves on the monopile. To this end, the monopile is placed in a flume with a closed end at one side, and a wave maker at the other. The wave maker consists of a paddle that can move horizontally, causing a standing long wave in the flume. See the accompanying figure.



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The flume has a length ℓ of 15 m, and an undisturbed depth of 35 cm. The monopile is placed at a distance of $\frac{3}{5}\ell$ from the wave maker. By changing the frequency and the amplitude of the horizontal stroke of the paddle, the wave height at the monopile can be adjusted.

IV.4$^{1.0}$ Use linear wave theory to calculate the smallest frequency for which the surface elevation amplitude of the standing wave in the flume has a maximum (anti node) at the monopile.

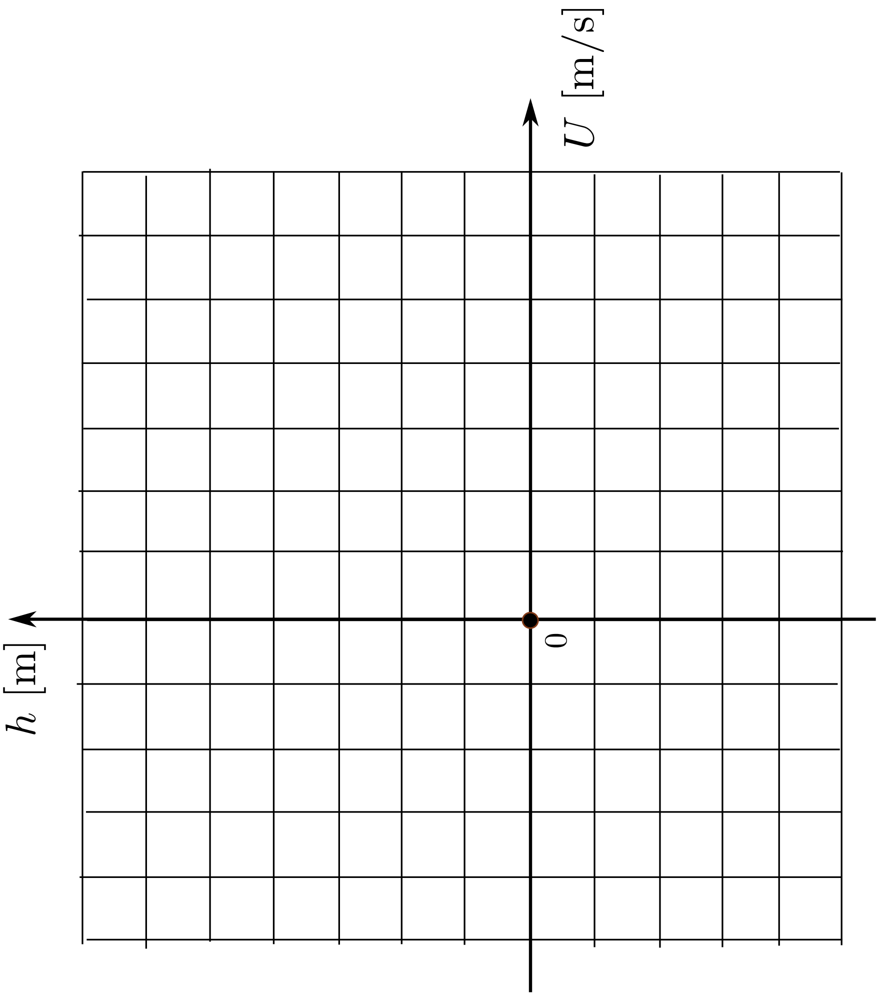
For a particular test, the surface elevation amplitude $\hat{\zeta}$ at the monopile corresponding to this frequency should be 5 cm.

IV.5$^{1.5}$ Use linear wave theory to calculate the velocity amplitude $\hat{U}_{\mathrm{st}}$ of the standing wave and the amplitude $\hat{a}$ of the horizontal stroke of the wave paddle in order to generate the specified wave height at the location of the monopile for the frequency calculated in Question IV.4.

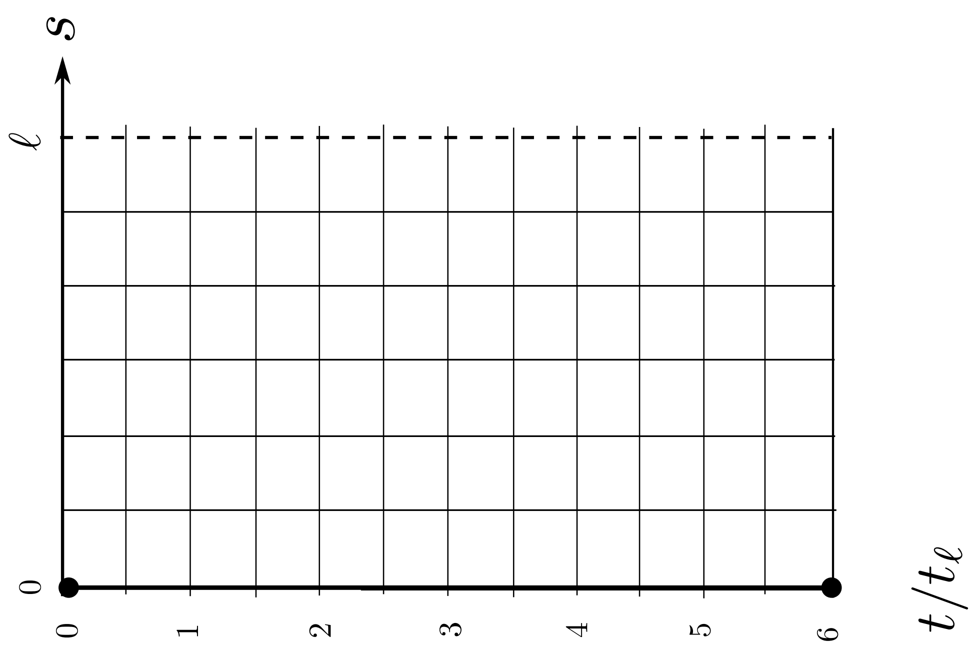
Answer form Question II.3

name:.......................................

student number:......................................



U, h-state diagram



s, t-plane

Answers

Question I (tsunami)

I.1$^{1.0}$ A long wave is characterized by the vertical pressure distribution being hydrostatic, which is the case if the curvature of the streamlines in the vertical plane is negligible. A criterion for this to hold is that the length scale $\mathcal{L}$ of a wave is much larger than the water depth by, say, a factor of 20. In the given case, the ratio of the horizontal length scale ($\mathcal{L} \approx L_0$) and the depth (d_0) equals about 135 which clearly satisfies the criterion.

I.2$^{1.0}$ In the give case the following three simplifications apply:

1. in the momentum equation, resistance can be neglected with respect to the local acceleration term because the time scale $\mathcal{T}$ of the problem is relatively small (water level increases instantaneously);
2. the advective acceleration term can be neglected with respect to the local acceleration term, a ratio which is proportional to the Froude number which in turn equals the wave height to depth ratio which in the given case is of the order of 10^{-3};
3. variations of the geometric parameters (the conveyance depth in this case) can be ignored because the surface level variations are negligibly small relative to the water depth.

I.3$^{1.5}$ The initial disturbance is split into two components, that will travel in opposite directions wave speeds $c = \pm\sqrt{gd_0} = \pm 121.3\,\mathrm{m/s}$. These two components have surface elevations $\zeta^{\pm} = \frac{1}{2}\left(\zeta_0 \pm (c/g)U_0\right)$. Since $U_0 = 0$, we obtain $\zeta^+ = \zeta^- = \frac{1}{2}\zeta_0$, which for both components amounts to 1.5 m in the region of the earthquake and zero elsewhere. The corresponding velocity disturbances are given by $U^{\pm} = \pm(g/c)\zeta^{\pm}$, respectively, which equals $\pm$ 0.12 m/s in the region of the earthquake and zero elsewhere. At the considered time instance, these components have travelled distances Δs of $\pm c\Delta t = \pm$ 72.6 km. The resulting surface elevation and flow velocity are obtained by adding the respective solutions at time $t = 600\,\mathrm{s}$. See the accompanying figure for the plots and corresponding numerical values.

I.4$^{1.0}$ The quantity L/c is the wave period, or total duration of the wave. It is constant for each wave component since the leading edge and trailing edge of the wave pass a given sequence of points in the same time interval. This is so because in a linear wave model the wave speed depends on the undisturbed depth only. The elapsed time between the passage of the leading edge and the trailing edge is then constant for every point. The quantity $\rho g\, L\, \zeta^2$ is the total energy in the wave (potential and kinetic) per unit width. If no wave energy is dissipated (see also Question I.2) this quantity remains constant during propagation, which is the basis of Green's law.

I.5$^{1.5}$ We only have to take into account the wave component that travels towards the coast (ζ^+) since the other component travels away from the coast and has a height equal to zero at the specified location. Since L is proportional to c (because L/c is constant) it follows that $\rho g c \zeta^2$ is constant. This gives $\rho g\,\sqrt{gd_1}\zeta_1^2 = \rho g\,\sqrt{gd_0}\left(\zeta_0^+\right)^2$ from which it follows that $\zeta_1 = \zeta_0^+\left(d_0/d_1\right)^{1/4}$. Using $\zeta_0^+ = 1.5\,\mathrm{m}$ this gives $\zeta_1 = 4.74\,\mathrm{m}$. The corresponding flow velocity is given by $U_1 = (g/c_1)\zeta_1 = 3.84\,\mathrm{m/s}$ (using $c_1 = 12.12\,\mathrm{m/s}$).

At the given location the wave height to depth ratio is large (≈ 0.35), so the neglect of the advective acceleration and the use of a linearized depth are not valid anymore. The results will therefore deviate from the actual situation. (An exact computation using $c = 3\sqrt{g(d_1+\zeta_1)} - 2\sqrt{gd_1}$ - see book Eqn. 4.17 - gives $\zeta_1 = 4.04\,\mathrm{m}$, so the approximate answer is actually not too bad!)

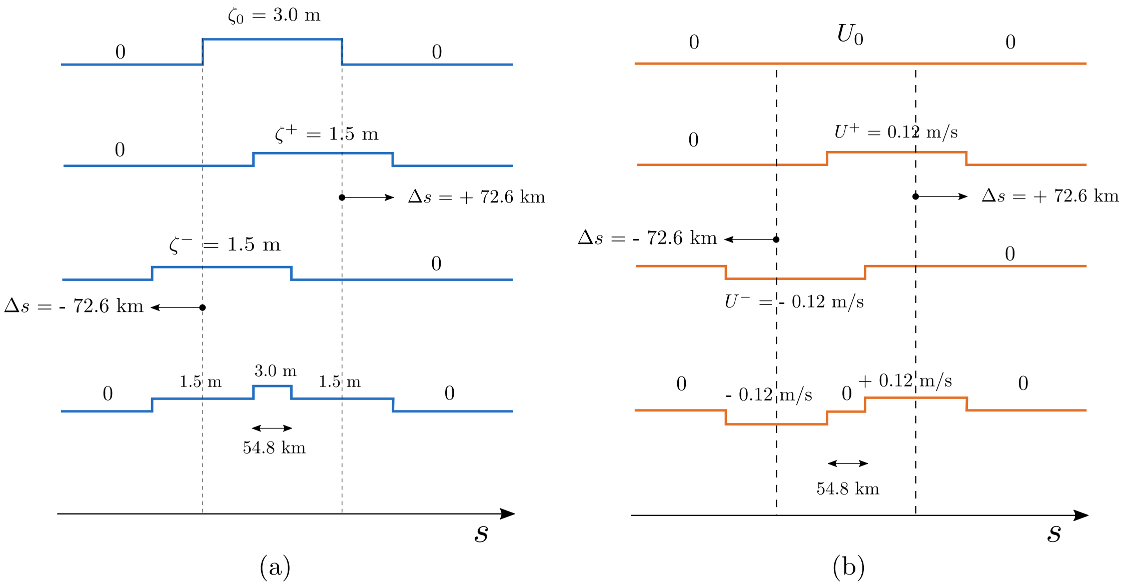


Figure 1: Question I.4: surface elevation (a) and flow velocity (b) at $t = 600\,\mathrm{s}$

Question II (fresh water intake)

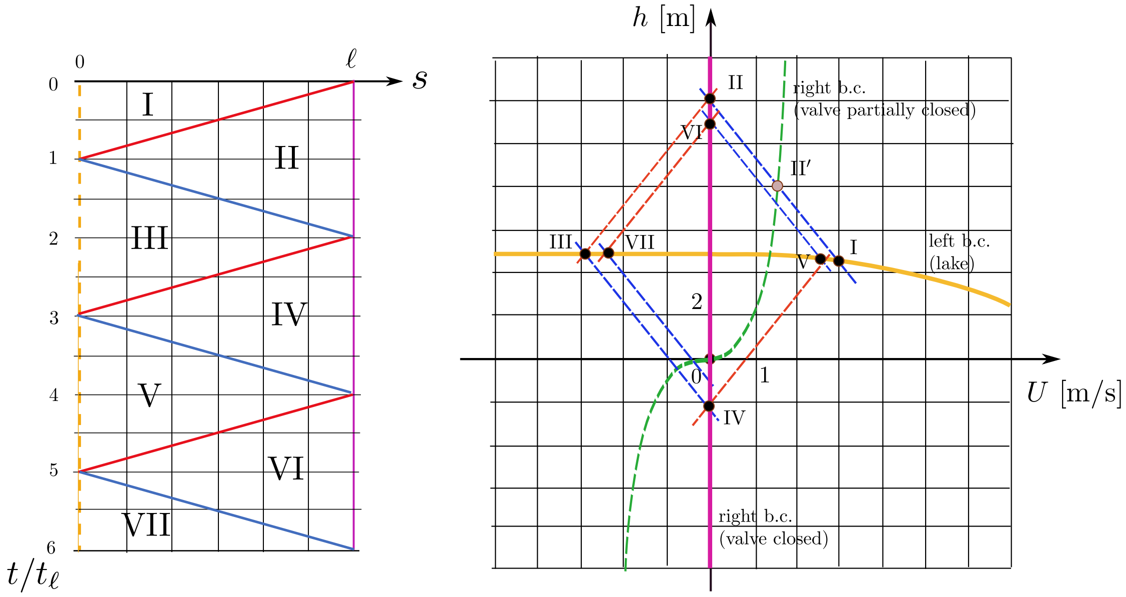
II.1$^{1.0}$ The propagation speed of pressure waves in a closed pipe line is given by $c^{-1} = \sqrt{\rho/K + \rho D/E\delta}$. For an inifinitely rigid pipe wall ($E\delta/D \to \infty$) the second term in the square root goes to zero, so that the resulting wave speed becomes $c = \sqrt{K/\rho} = 1480\,\mathrm{m/s}$. Variations of the piezometric level and the velocity are related by $\delta h^{\pm} = \pm(c/g)\delta U^{\pm}$ (for right and left travelling waves, respectively). Due to the very high propagation speed of pressure waves in general, a small velocity variation already results in a large pressure variation which has to be compensated by a large normal stress in the pipe wall that could exceed the critical stress. This in particular holds for an inifinitely rigid pipe for which the wave speed is higher than for an elastic pipe (elasticity of the pipe wall reduces the wave speed), making them prone to failure.

II.2$^{1.0}$ The neglect of wall friction and steady flow leads to h_0 and U_0 being uniform in the pipe. At the inflow end we have $h_0 + U_0^2/2g = h_{\text{lake}}$ (zero energy loss), and at the outflow end $h_0 - h_{\text{res}} = \xi U_0^2/2g$ (discharge relation valve). Elimination of h_0 leads to $U_0 = \sqrt{2g(h_{\text{lake}} - h_{\text{res}})/(\xi + 1)}$. For $\xi = 9$ and the given water levels in the lake and reservoir this gives $U_0 = 3.13\,\mathrm{m/s}$. Substitution of this value in one of the relations at the inflow - or outflow end gives $h_0 = 4.50\,\mathrm{m}$.

II.3$^{2.0}$ See the accompanying figure.

II.4$^{1.0}$ The highest piezometric level in the pipe after closing the valve corresponds to state II in the U, h-state diagram. It occurs for $t = 0$ at the valve ($s = \ell$) and propagates away from the valve - in negative s-direction - until it covers the entire pipe at time $t = t_\ell$. Reflection of a negative wave at $s = 0$ - which propagates in positive direction - gradually eliminates the maximum piezometric level maximum to become h_{lake}, uniformly, at time $t = 2t_\ell$. State II results from the intersection of the characteristic relation (positive characteristic) with the vertical axis (boundary condition $U = 0$ at $s = \ell$). For a partially closed valve the latter would become the right branch of a parabola representing the discharge relation of the valve. Consequently, this would give a smaller value of h_{II} (state II' in the accompanying figure).

II.5$^{1.5}$ The (new) state satisfies the characteristic relation along the corresponding positive characterictic: $h_{II} + (c/g)U_{II} = h_I + (c/g)U_I$. It also satisfies the boundary condition at the valve: $h_{II} = h_{\text{res}} + \xi U_{II}^2/2g$, with ξ yet unknown. Since $h_{II} = h_{\max} = 40\,\mathrm{m}$, it follows from the first equation that $U_{II} = 2.90\,\mathrm{m/s}$. Substitution of h_{II} and U_{II} in the discharge relation of the valve gives finally $\xi = 93.5$.



(a) (b)

Figure 2: Question II.3: (s,t)-plane (a) and (U,h)-state diagram (b); characteristic relations are plotted schematically, not representing the correct slope

Question III (tidal river)

III.1$^{1.0}$ The two dimensionless parameters that the determine the principal features of tidal wave propagation in a tidal river are the basin length to wave length ratio ℓ/L_0 (or, alternatively, $k_0\ell$), and the relative resistance $\sigma = \kappa/\omega$. If the main channel is deepened the wave speed c_0 increases, giving a larger wave length and consequently a smaller basin to wave length ratio. The relative resistance is inversely proportional to the depth (because $\kappa = c_f\hat{U}/R$) and will likely decrease if the channel is deepened, provided that this reduction is not overruled by an increase of the velocity $\hat{U}$ (which in most practical cases is however unlikely).

III.2$^{1.5}$ The response factor $r = \hat{\zeta}_\ell/\hat{\zeta}_0 = 1.5$, while $k_0\ell = 1.0$ (compute this using $c_0 = \sqrt{gA_c/B} = 9.13\,\mathrm{m/s}$ and $k_0 = \omega/c_0 = 1.53\times10^{-5}\,\mathrm{rad/m}$). From the graph can be seen that for $k_0\ell = 1$ the response factor equals 1.5 for $\sigma = 1$, corresponding to $\kappa = \omega\sigma = 1.4\times10^{-4}\,\mathrm{s}^{-1}$.

III.3$^{1.5}$ The amplitude of the dicharge at the mouth of the tidal river is given by $\hat{Q}_0 = Bc\cos\delta\,\hat{\zeta}_\ell\sqrt{\sinh^2\mu\ell + \sin^2 k\ell}$. (Note that $\hat{\zeta}_\ell$ has been used instead of $\hat{\zeta}_0$ in order to simplify the formula.) To compute it, first compute the resistance angle $\delta = \frac{1}{2}\arctan\sigma = 0.39\,\mathrm{rad}$ (22.5°), the wave number $k = k_0\sqrt{1-\tan^2\delta} = 1.69\times10^{-5}\,\mathrm{rad/m}$, and the damping parameter $\mu = k\tan\delta = 6.98\times10^{-6}\,\mathrm{s}^{-1}$. Next, the wave speed is calculated, $c = \omega/k = c_0\sqrt{1-\tan^2\delta} = 8.31\,\mathrm{m/s}$, and finally the discharge amplitude $\hat{Q}_0 = 18{,}520\,\mathrm{m^3/s}$.

III.4$^{1.0}$ The linear resistance factor κ depends on the amplitude of the velocity $\hat{U}$ which is a part of the solution and can not be determined beforehand. To proceed, an unitial guess for κ has to be assumed (for instance based on the old situation) for which the problem is solved. This yields an provisional value for $\hat{U}$ which is used to improve the estimate for κ. The process is repeated, in an iterative manner, until the solution has converged.

III.5$^{1.0}$ The computation involves the calculation of $c_0 = 9.79\,\mathrm{m/s}$ using the new conveyance area, the wave number $k_0 = \omega/c_0 = 1.43\times10^{-5}\,\mathrm{rad/m}$, and the relative resistance $\sigma = \kappa/\omega = 0.82$ followed by $\delta = 0.34\,\mathrm{rad}$. Using the new value for δ, the wavenumber $k = 1.53\times10^{-5}\,\mathrm{rad/m}$, the damping factor $\mu = 5.49\times10^{-6}\,\mathrm{m}^{-1}$, and the wave speed $c = \omega/k = 9.13\,\mathrm{m/s}$. Everything is now available to compute the discharge $\hat{Q}_0 = Bc\cos\delta\,\hat{\zeta}_0\sqrt{\sinh^2\mu\ell + \sin^2 k\ell}/\sqrt{\sinh^2\mu\ell + \cos^2 k\ell}$, using $\hat{\zeta}_0 = 0.80\,\mathrm{m}$ ($\hat{\zeta}_\ell$ is not known this time) giving, finally, $\hat{Q}_0 = 19{,}240\,\mathrm{m^3/s}$, an increase $\Delta\hat{Q}_0$ of $720\,\mathrm{m^3/s}$. Alternative approach: since

$\kappa = c_f \hat{Q}_{\text{rep}}/A_c R$, where $\hat{Q}_{\text{rep}}$ is the so-called representative discharge amplitude in the tidal river. The latter is proportional to the discharge at the mouth which, using that c_f does not change, leads to the following proportionality: $\kappa \propto \hat{Q}_0/A_c R$, which can also be written as $\hat{Q} \propto \kappa A_c R$. In the new situation, κ becomes $1.15/1.4 = 0.92$ times larger, while A_c and R both become 1.15 times larger. The discharge amplitude therefore becomes $0.92 \times 1.15 \times 1.15 = 1.083$ times larger, 21,868 m^3/s. (This result is slightly different from the previous answer as the resistance factor κ has been rounded.)

Question IV (wave tests monopile)

IV.1$^{1.0}$ The speed with which a disturbance propagates is given by $U + c$. The celerity c depends on the local depth which for a high translatory wave is larger at the trailing edge than at the leading edge of the wave. On top of this, the flow velocity U is higher at the trailing edge of the wave. Points with a larger depth therefore travel faster causing the wave to steepen and ultimately break. The terms in the shallow water equations that account for this are the nonlinear momentum advection term ($U^2 d$) and the nonlinear dependence of the pressure force on the depth ($\frac{1}{2} g d^2$).

IV.1$^{1.0}$ See the accompanying figure. The positive characteristics corresponding to the compression wave are straight converging lines. They are straight because not only the quantity $U + 2c$ is constant along them (by definition), but also the quantity $U - 2c$ which is constant in the entire domain (every negative charateristic is issued from the rest state). Therefore, the flow state is constant along a positive charateristic, and so is the characteristic speed $U + c$ implying that these characteristics are straight lines. Because in a certain time interval the inflow velocity increases, the local depth and flow velocity increase, which incraeses the value of the characteristic speed $U + c$ at the inflow boundary over time. The characteristics are therefore converging lines. The first intersection point in the s, t plane indicates the time instant and location where the compression wave will break, which is determined - and can be controlled - by the rate with which the inflow velocity increases in time.

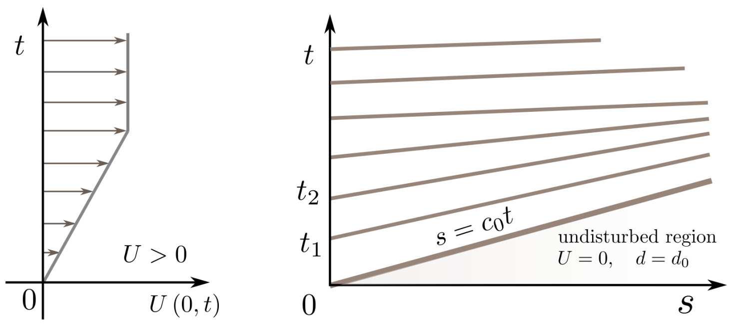


Figure 3: Compression wave: velocity variation at inflow boundary (left) and (s, t) diagram showing the positive characteristics (right)

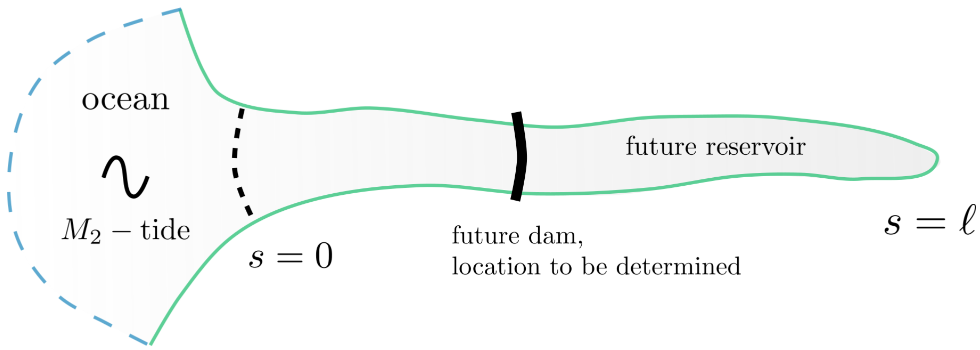
IV.3$^{1.5}$ The specific discharge of the bore is related to the height of the bore by $q = c\,(d_1 - d_0)$, where c depends on both d_1 and d_0 by the given expression. For a given specific discharge q and undisturbed depth d_0, the depth d_1 can be computed by solving these two equations iteratively; use an initial guess for d_1, for instance $d_1 = d_0$, to compute c. This gives an improved estimate $d_1 = d_0 + q/c$. The procedure is now repeated until convergence (usually within very few iterations). the final result is $d_1 = 0.44$ m, and the wave height $d_1 - d_0 = 0.09$ m.

IV.4$^{1.0}$ At the close end we have an anti node. There will be another node at the monopile if the distance between the monopile and the closed end equals a whole number of half wave lengths. The case with half a wave length corresponds to the smallest frequency (largest wave length), and has a wave length determined by $\frac{1}{2}L = \frac{2}{5}\ell$. It follows that $L = \frac{4}{5}\ell = 12$ m, and the period $T = L/\sqrt{g d_0} = 6.48$ s (using $c_0 = 1.85$ m/s). The frequency, finally, equals $\omega = 2\pi/T = 0.97$ rad/s.

IV.5$^{1.5}$ Because we have an anti node at the monopile, the surface elevation amplitude at the monopile is also the surface elevation amplitude of the standing wave, so $\hat{\zeta}_{\rm st} = 5\,\text{cm}$. This gives a velocity amplitude of the standing wave $\hat{U}_{\rm st} = (g/c)\hat{\zeta}_{\rm st} = 0.26\,\text{cm/s}$. The velocity amplitude varies sinusoidally being 0 at the close end and $\hat{U}_0 = \hat{U}_{\rm st}|\sin k\ell|$ at the wave maker. Using $k\ell = (2\pi/\frac{4}{5}\ell)\ell = \frac{5}{2}\pi$ ($\sin k\ell = 1$), it follows that $\hat{U}_0 = \hat{U}_{\rm st} = 0.26\,\text{m/s}$. Since $\hat{U}_0 = \omega\hat{a}$, we finally obtain $\hat{a} = 0.27\,\text{m}$.

Question I (tidal river with reservoir) - *use a new answer form*

A tidal river has an open connection to the ocean. Anticipating the increased occurrence of extremely dry periods, a part of the river will be turned into a fresh water reservoir. The reservoir is created by closing off a part of the river with an impermeable dam of which the optimal location has yet to be determined. To this end, some preliminary tidal computations are made using the harmonic method. See also the accompanying figure.



I.1$^{1.0}$ Explain which assumptions and simplifications are involved in the linearization of the one-dimensional shallow water equations for tidal problems.

The tidal river is schematized as a prismatic channel having a width $B = 2\,B_c$ of 1,200 m, a depth d of 8 m, a length ℓ of 75 km, and a resistance factor κ of $1 \cdot 10^{-4}\,\mathrm{s}^{-1}$. At the ocean boundary ($s = 0$) an M_2-tide prevails with a surface level amplitude of 1.25 m.

I.2$^{1.5}$ Compute the surface level amplitude at the closed end of the tidal river ($s = \ell$) for the original situation - before the construction of the dam.

I.3$^{1.0}$ Interpret the result of Question I.2 by calculating the two dimensionless parameters that determine the response of the tidal system.

The optimal location of the dam depends on two criteria: 1) the storage capacity of the resulting reservoir should be sufficient, and 2) the surface level amplitude in the remaining part of the tidal river should be maximal. It is argued that the second requirement is largely met if the ratio of the basin length and the actual wave length L (including the effect of resistance) equals $\frac{1}{4}$.

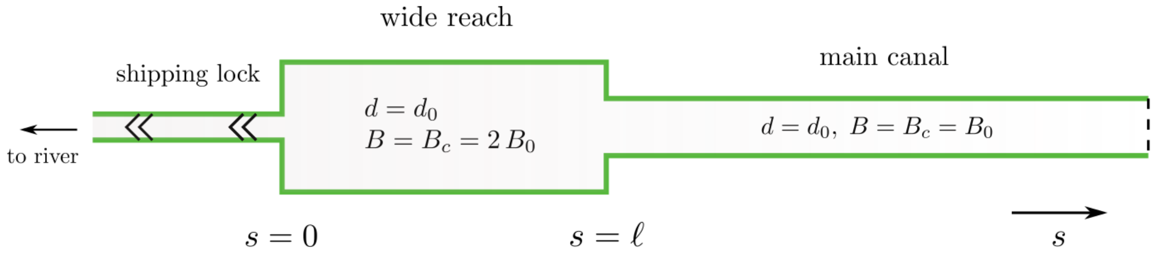
I.4$^{1.5}$ For a basin - wave length ratio of $\ell/L = \frac{1}{4}$, compute the corresponding surface elevation amplitude at the closed end of the remaining part of the tidal river (at the dam), assuming that all parameters other than the length of the tidal river remain unchanged.

During dry periods there is no inflow of fresh water into the reservoir while a daily amount V of $280 \cdot 10^6$ litres is extracted for agriculture and human consumption.

I.5$^{1.0}$ For the reservoir corresponding to the situation of Question I.4, compute the rate with which the reservoir level drops during dry periods.

Question II (partial reflection near shipping lock) - *use a new answer form*

A shipping canal is connected to a river by means of a lock. In order to accommodate the manoeuvring of ships into and out of the lock, the canal reach adjacent to the lock has a width twice that of the main canal. This involves an abrupt change of the canal width at a certain distance ℓ from the lock from which translatory waves, caused by levelling the lock, are partially reflected. Between levelling operations, a state of rest prevails in the canal with a water level $h = 0$ and flow velocity $U = 0$. See the accompanying figure.



A levelling procedure starts at time $t = 0$, from which the water level in the lock is lowered with a constant rate of 60 cm/min. The lock chamber has a wet surface area of $20 \times 100\,\mathrm{m}^2$. The canal has an undisturbed depth d_0 of 2.5 m and the main canal width B_0 (see figure) equals 40 m.

II.1$^{1.0}$ Calculate the height and flow velocity of the translatory wave that is issued from the lock at time $t = 0$; verify that this wave can be considered a linear wave.

Some time after the levelling has started, the translatory wave issued from the lock will reflect partially from the transition at $s = \ell$.

II.2$^{1.0}$ Explain the principle that is used to formulate the effect of an abrupt canal transition on the propagation of low translatory waves.

The wave that is initially reflected from the transition will travel back towards the lock, where it is reflected again, after which it is reflected once more at the transition, and so on. The accumulated effect of these multiple reflections will be studied by applying the method of characteristics to the canal reach between $s = 0$ and $s = \ell$. In absence of waves from the main canal reach, the corresponding boundary condition at $s = \ell$ can be formulated as follows

$$h\,(\ell) = \gamma^{-1}\,\frac{c_0}{g}\,U\,(\ell)\,,$$

in which γ is the ratio of the Bc-value of the main canal to that of the wider reach, respectively.

II.3$^{1.0}$ Demonstrate that the above formulation of the boundary condition at $s = \ell$ includes the cases of fully positive and fully negative reflection as well as the so-called absorbing boundary, depending on the value of γ.

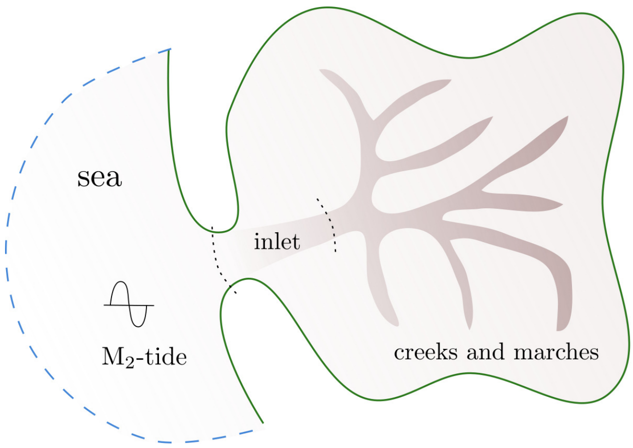
The time interval of interest is $0 < t \leq 5\,t_\ell$, where t_ℓ is the travel time of waves in the considered canal reach. During this time interval the boundary condition for the velocity at $s = 0$ remains constant.

II.4$^{3.0}$ Construct the graphical solution to the above problem for the considered time interval using the characteristic equations $\mathrm{d}h/\mathrm{d}U = \mp\,c_0/g = \mp\,0.5\,\mathrm{m/(m/s)}$ along $\mathrm{d}s/\mathrm{d}t = \pm\,c_0$. Use the accompanying answer form; plot the initial - and boundary conditions using the appropriate value of γ for the boundary condition at $s = \ell$.

II.5$^{1.0}$ Indicate which state occurs at $s = \ell$ shortly after the first occurrence of reflection at the transition, and calculate the corresponding values of the water level and flow velocity - these may also be derived graphically, provided your plot is sufficiently accurate.

Question III (coastal wetland) - *use a new answer form*

As a part of a nature rehabilitation project a coastal wetland will be created. The wetland consists of a number of shallow creeks - feeding the tidal marches that surround them - which are connected to the sea by means of a natural inlet. At sea an M_2-tide prevails, causing tidal variations in the wetland, which is crucial to establish certain types of wildlife. See the accompanying figure for a conceptual sketch of the intended situation.



The most important element of the design is the surface level amplitude inside the wetland which, apart from the tide at sea, depends on the wet surface area of the creeks and marshes in relation to the properties of the inlet. For this wetland, it can be assumed that all its components are small relative to the tidal wave length, and that the role of inertia in the resulting tidal system is negligible.

III.1$^{1.0}$ Explain how the conveyance cross section A_c of the inlet and the generalized resistance factor χ influence the surface elevation amplitude inside the wetland by arguing whether the amplitude will increase or decrease if only the respective parameter is varied.

The creeks and marches have a total wet surface area A_b of $8.0\,\mathrm{km}^2$. At sea, the surface elevation amplitude $\hat{\zeta}_\mathrm{s}$ amounts to $0.65\,\mathrm{m}$. Inside the wetland, the surface elevation amplitude $\hat{\zeta}_\mathrm{b}$ will be $0.35\,\mathrm{m}$ to create optimal conditions for wildlife. The parameters A_c and χ of the inlet are to be chosen such that these tidal conditions inside the wetland are realized.

III.2$^{1.5}$ For an inlet which is designed as a short gap (i.e. with zero length), compute the required conveyance cross section A_c.

III.3$^{1.0}$ For the situation corresponding to Question III.2, compute the velocity amplitude at the inlet and the phase lag of the tidal water level inside the wetland.

From nearby wetlands it is known that the inlet, which mainly consists of sand, is morphologically stable only if the tidal velocity amplitude $\hat{U}_\mathrm{eq}$ at the inlet equals $0.95\,\mathrm{m/s}$. Clearly, according to this criterion, the inlet from Question III.2 is unstable.

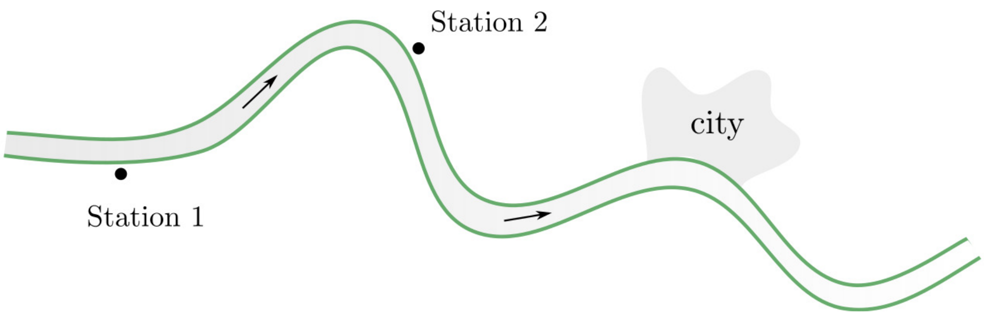
III.4$^{1.5}$ Compute the conveyance cross section $A_{c,\mathrm{eq}}$ for which the inlet will be stable and, next, compute the corresponding resistance parameter χ giving the required tide inside the wetland.

The inlet channel from Question III.4 will have a hydraulic radius R of $2.0\,\mathrm{m}$ and a resistance coefficient c_f of $5 \cdot 10^{-3}$.

III.5$^{1.0}$ Calculate the resulting length ℓ of the inlet, and verify that inertia can still be ignored in the resulting tidal wetland system (so that the above computations are in fact correct).

Question IV (high-water warning) - *use a new answer form*

Following a sudden, heavy rainfall event in the catchment area of a river, a high-water wave approaches a large city. The behaviour of this wave is observed closely in order to take timely and adequate precautions. To this end, the peak arrival times and wave heights are recorded at upstream Stations 1 and 2, respectively, which can be used to predict the arrival time and wave height at the city. See the accompanying figure.



The predictions involve some calculations using a linearized model for high-water wave propagation in rivers. To this end, the river is schematized using a uniform width $B = 3B_c = 600\,\text{m}$, a uniform bed slope $i_b = 2 \cdot 10^{-4}$, and a uniform resistance coefficient $c_f = 4 \cdot 10^{-3}$. The peak of the wave travels in 12 hrs from Station 1 to Station 2, which is a streamwise distance of 35 km.

IV.1[1.0] Compute the speed of the high-water wave during its passage from Station 1 to Station 2, and compute the corresponding water depth in the river (using $d \approx R$).

It is observed that during the passage of the high-water wave from Station 1 to Station 2, the wave height changes considerably.

IV.2[1.0] Explain the physical mechanism that causes this change.

The rainfall event can be schematized as an instantaneous release of a large water volume at an upstream point of the river, resulting in a Gaussian high-water wave that gradually spreads and collapses as it travels downstream. When the peak of this wave passes Station 2, the wave has a standard deviation σ of 25 km. The streamwise distance between Station 2 and the city amounts to 45 km.

IV.3[1.5] Compute the standard deviation of the high-water wave when its peak arrives at the city. (Hint: σ^2 increases linearly in time.)

The observed wave height when the peak passes Station 2 amounts to 2.75 m

IV.4[1.5] Compute the total water depth at the city when the wave peak passes.

The water level observations at Stations 1 and 2 also show several small disturbances that appear before and after the passage of the bulk of the high-water wave.

IV.5[1.0] Explain the origin and behaviour of these disturbances.

Answer form Question II.4

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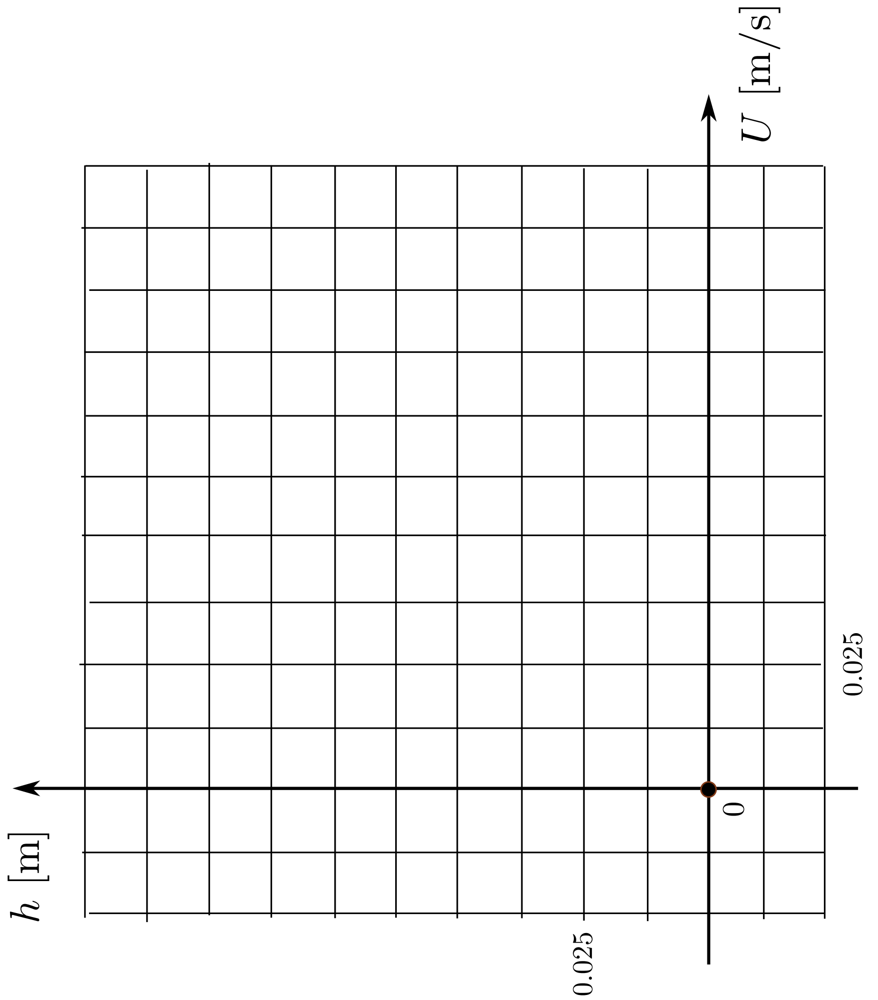
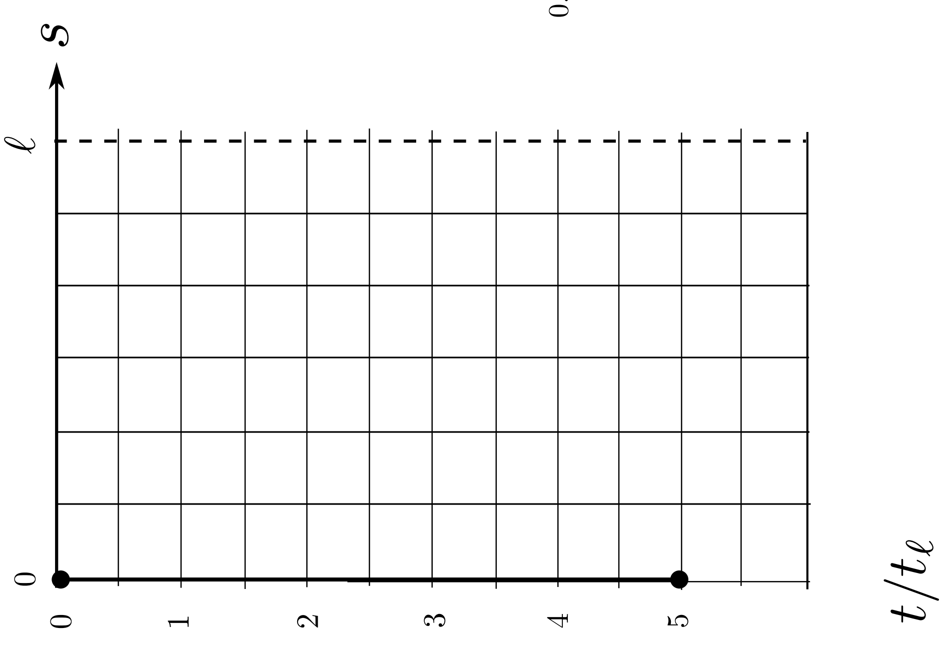


U, h-state diagram



s, t-plane

Answers

Question I (tidal river with reservoir)

I.1$^{1.0}$ To linearize the shallow water equations every term should depend linearly on the solution only. This involves three aspects:

1. the cross-sectional parameters are independent on the solution, which is approximately the case if the water level variation (i.e. the wave height) is small relative to the water depth;
2. the advective acceleration term is small relative to the local acceleration term, which requires the Froude-number to be small ($\ll 1$), which is the case when the wave height to depth ratio is small (this complies with the first condition);
3. the quadratic resistance term must be replaced with an equivalent linear resistance term, such that its energy dissipation during a tidal cycle equals that of the quadratic resistance (Lorentz).

I.2$^{1.5}$ First calculate the wave parameters in absence of resistance: wave speed $c_0 = \sqrt{gA_c/B} = 6.26\,\mathrm{m/s}$, wave number $k_0 = \omega/c_0 = 2.23 \cdot 10^{-5}\,\mathrm{rad/m}$. Next, calculate the relative resistance $\sigma = \kappa/\omega = 0.71$, and the resistance angle $\delta = \frac{1}{2}\arctan\sigma = 0.31\,\mathrm{rad}$. This gives a wave number $k = k_0/\sqrt{1-\tan^2\delta} = 2.36 \cdot 10^{-5}\,\mathrm{rad/m}$, and damping coefficient $\mu = k\tan\delta = 7.56 \cdot 10^{-6}\,\mathrm{m}^{-1}$. The amplitude at the closed end ($s = \ell$) is now given by $\hat{\zeta}_\ell = \hat{\zeta}_0/\sqrt{\sinh^2\mu\ell + \cos^2 k\ell} = 1.99\,\mathrm{m}$ (amplification factor $r = 1.61$).

I.3$^{1.0}$ The dimensionless parameters that determine the response of the tidal river are the relative resistance $\sigma = 0.71$ and the basin length to wave length ratio $\ell/L_0 = 0.27$. The latter is pretty close to the resonant value of $\frac{1}{4}$ (quarter wave-length resonance) indicating a large amplitude ratio, while the relative resistance is significant, but not dominating the problem. We therefore expect a large amplitude ratio, albeit not extremely large due to the influence of the resistance.

I.4$^{1.5}$ For $\ell/L = \frac{1}{4}$ (accounting for resistance) we have $k\ell = \frac{1}{2}\pi$. This gives for the surface level amplitude at the closed end: $\hat{\zeta}_\ell = \hat{\zeta}_0/|\sinh\mu\ell|$. Using $\mu\ell = k\ell\ \tan\delta = \frac{1}{2}\pi\ \tan\delta = 0.50$, we obtain $\hat{\zeta}_\ell = 2.38\,\mathrm{m}$.

I.5$^{1.0}$ The length of the remaining part of the tidal river can be computed using $k\ell = \frac{1}{2}\pi$, giving $\ell = 66.58\,\mathrm{km}$. This gives a reservoir length ℓ_{res} of $8.42\,\mathrm{km}$, and a wet surface area of the reservoir of $A_{\mathrm{res}} = \ell_{\mathrm{res}}\,B = 10.1\,\mathrm{km}^2$. The rate with which the water level in the reservoir decreases during dry periods is now given by $V/A_{\mathrm{res}} = 2.8\,\mathrm{cm/day}$.

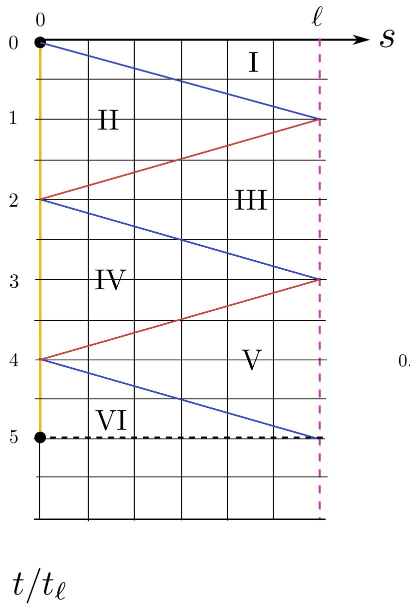
Question II (partial reflection near shipping lock)

II.1$^{1.0}$ The discharge released from the lock equals $Q = \mathrm{d}h/\mathrm{d}t|_{\mathrm{lock}}\,A_b = 20.0\,\mathrm{m}^3/\mathrm{s}$. This results in a translatory wave propagating in the positive $s-$direction with a wave height $\delta h^+ = Q/Bc = 0.05\,\mathrm{m}$ (using $B = 2B_0 = 80\,\mathrm{m}$, and $c_0 = \sqrt{gd_0} = 5.0\,\mathrm{m/s}$). The flow velocity corresponding to this wave equals $\delta U^+ = (g/c_0)\delta h^+ = 0.10\,\mathrm{m/s}$. The wave height to depth ratio equals $\delta h^+/d_0 = 0.05/2.5 = 0.02 \ll 1$, which justifies the application of linear wave theory.

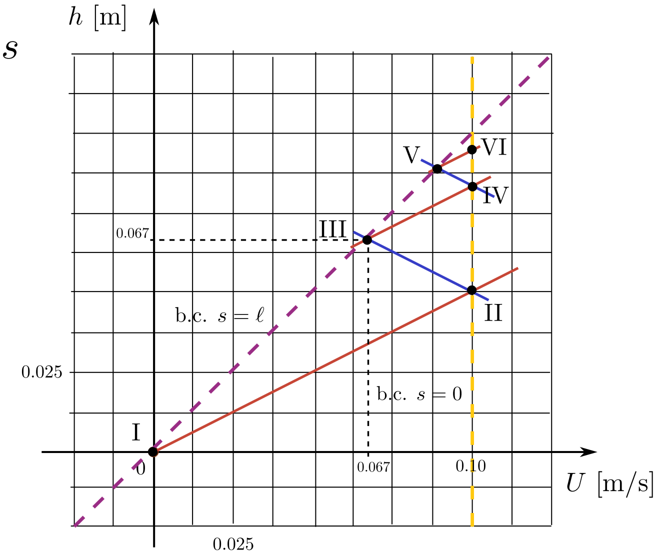
II.2$^{1.0}$ For linear waves, the water level and discharge at an abrupt transation of the cross section are assumed continuous. The sum of the heights of the incoming and reflected wave is then equal to the height of the transmited wave, with a similar relation for the corresponding discharges. These two conditions can be expressed in terms of two coefficients, one for the reflected and one for the transmitted wave height, giving two equations with two unknowns. If the waves are nonlinear, the discharge is still continuous at the transition while the water level is not, a result of effects related to the velocity head which are ignored in the linear approximation.

II.3$^{1.0}$ For $\gamma \to \infty$ (infinitely wide main canal, or reservoir) the boundary conditions reads $h(\ell) = 0$, corresponding to fully negative reflection. For $\gamma \downarrow 0$ (infinitely narrow main canal, or closed wall) the boundary condition reads $U(\ell) = 0$, corresponding to fully positive reflection. For $\gamma = 1$ (uniform canal) the boundary condition reads $h(\ell) - (c_0/g)U(\ell) = 0$ stating that the Riemann variable for the ingoing characteristic equals zero, corresponding to an absorbing boundary condition (a for a simple wave).

II.4$^{3.0}$ The initial conditions are $h_I = 0$ and $U_I = 0$. The boundary condition at $s = 0$ is given by $U(0) = Q/(2B_0 d_0) = 0.10\,\mathrm{m/s}$. The value of γ equals $(B_0 c_0)/(2B_0 c_0) = 0.5$, with the boundary condition at $s = \ell$ given by $h(\ell) = 2(c_0/g)U(\ell) = U(\ell)$. See the accompanying figure.



s, t-plane



U, h-state diagram

II.5$^{1.0}$ The state at $s = \ell$ directly after the first occurrence of relection at this location corresponds to state III in the s,t-plane. From the state diagram it can be inferred that $h_{III} - h_{II} = -(c_0/g)\,(U_{III} - U_{II})$ (characteristic relation along positive characteristics), and that $h_{III} = 2(c_0/g)U_{III}$ (boundary condition at $s = \ell$). elimination of U_{III} gives $\frac{3}{2}h_{III} = h_{II} + (c_0/g)U_{II}$ where state II corresponds to the translatory wave issued from the lock as the levelling starts (see Question II.1). This gives $h_{III} = 0.067\,\mathrm{m}$ and $U_{III} = 2(c_0/g)h_{III} = 0.067\,\mathrm{m/s}$.

Question III (coastal wetland)

III.1$^{1.0}$ If the resistance factor χ decreases the tidal damping in the considered system decreases leading to a larger surface level amplitude inside the basin. For a smaller conveyance cross section A_c of the inlet, the same discharge causes a larger flow velocity at the inlet and an increased head loss, leading to increased damping and a reduction of the surface level amplitude in the basin. The same conclusion can be obtained by considering the resistance paraters $\Gamma = \frac{8}{3\pi}\chi\,(\omega A_b/A_c)^2\,\hat{\zeta}_s/g$ or $\omega\tau = \Gamma r = \frac{8}{3\pi}\chi\,(\omega A_b/A_c)^2\,\hat{\zeta}_b/g$, which for a inlet-basin system without inertia solely determine the amplitude ratio. A deceasing χ means that Γ and $\omega\tau$ also decrease, leading to a larger value of r while a decrease of A_c increases Γ and $\omega\tau$ which reduces r.

III.2$^{1.5}$ The required value of the amplitude ratio $r = \hat{\zeta}_b/\hat{\zeta}_s = 0.54$. The corresponding value of Γ can be determine using $r = 1/\sqrt{1+(\Gamma r)^2}$, which gives $\Gamma = \sqrt{1-r^2}/r^2 = 2.91$. The required cross section A_c now follows from the definition of Γ using a resistance parameter $\chi = \frac{1}{2}$ for the short gap. This gives $A_c = 110\,\mathrm{m}^2$.

III.3$^{1.0}$ The tidal discharge amplitude at the inlet is given by $\hat{Q} = \omega A_b \hat{\zeta}_b = 393\,\mathrm{m}^3/\mathrm{s}$, resulting in a velocity amplitude of $\hat{U} = \hat{Q}/A_c = 3.56\,\mathrm{m/s}$. The phase lag θ of the tide in the wetland with respect to the tide at sea is given by $\theta = \arccos r = 1.0\,\mathrm{rad}$. Note that both these answers can be computed independently from Question III.1, since the required values of $\hat{\zeta}_b$ and r are aleady given.

III.4$^{1.5}$ Since the tide inside the wetland and the wetland itself do not change, the equilibrium cross-section of a stable inlet equals $A_{c,eq} = \hat{Q}/\hat{U}_{eq} = 413\,\mathrm{m}^2$. However, to keep the tide inside the wetland the same, the parameter Γ may not change. Since Γ is proportional to χ and inversely proportional to A_c^2 it follows that χ should be increased in such a way that χ/A_c^2 remains the same. This gives $\chi = \frac{1}{2}\left(A_{c,eq}/A_c\right)^2 = 7.0$.

III.5$^{1.0}$ The length ℓ of the inlet is chosen such that $\chi = \frac{1}{2} + c_f \ell/R$ equals the required value computed in Question III.4. Using the given values of R and c_f this gives $\ell = 2{,}606\,\mathrm{m}$. The corresponding natural (resonant) frequency of the inlet-wetland system is now given by $\omega_0 = \sqrt{g A_{c,eq}/\ell A_b} = 4.4 \cdot 10^{-4}\,\mathrm{rad/s}$. To determine the influence of inertia relative to that of the resistance consider the parameters $(\omega/\omega_0)^2 = 0.10 \ll 1$ and $\omega\tau = \Gamma r = 1.56 > 1$. It can therefore be concluded that the response of the system is still determined by the resistance only.

Question IV (high-water warning)

IV.1$^{1.0}$ The wave speed equals distance between Station 1 and Station 2 divided by the elapsed time: $c_{\mathrm{HW}} = \Delta s_{1\to 2}/\Delta t_{1\to 2} = 0.80\,\mathrm{m/s}$. From the linearized model, the analytical expression for the wave speed reads $c_{\mathrm{HW}} = \frac{3}{2}(B_c/B)U_u$, from which the uniform flow velocity is calculated as $U_u = 1.62\,\mathrm{m/s}$. Using $U_u = \sqrt{g R_u i_b / c_f}$ and $R = d$, it follows that $R_u = d_u = 5.35\,\mathrm{m}$.

IV.2$^{1.0}$ The change of the wave height change is due to the influence of the depth gradient on the discharge. In the leading branch of the wave this effect increases the discharge compared to the discharge or a uniform flow (zero depth gradient), while in the trailing branch of the wave it reduces the discharge relative to that for a uniform flow. As a result, the water volume between two cross sections having equal depth but situated in the leading and trailing edge of the wave, respectively, decreases. This implies a reduction of the wave height and an increase of the wave length.

IV.3$^{1.5}$ The standard deviation σ of a Gaussian high-water wave increases in time according to $\sigma = \sqrt{2Kt}$, where t is the elapsed time since the instantaneous release of the water volume and $K = Q_u/(2 i_b B)$ is a diffusion coefficient. Using $Q_u = U_u B_c d = 1{,}735\,\mathrm{m}^3/\mathrm{s}$ gives $K = 7{,}228\,\mathrm{m}^2/\mathrm{s}$. Using $\sigma^2 = 2Kt$, the standard deviation of the wave when its peak passes the city can be computed from $\sigma_{\mathrm{city}}^2 = \sigma_2^2 + 2K\Delta t$, where Δt is the travel time of the wave from Station 2 to the city: $\Delta t = \Delta s / c_{\mathrm{HW}} = 55{,}542\,\mathrm{s}$. This gives finally $\sigma_{\mathrm{city}} = 37{,}8\,\mathrm{km}$.

IV.4$^{1.5}$ The total water volume V in the high water wave remains constant from which it follows that $\Delta d_{\mathrm{peak}}\sigma$ remains constant. The peak height is therefore inversely proportional to the standard deviation of the Gaussian wave. We now obtain $\Delta d_{\mathrm{peak,city}} = \Delta d_{\mathrm{peak,2}} \times (\sigma_2/\sigma_{\mathrm{city}}) = 1.82\,\mathrm{m}$. The total water depth at the passage of the wave peak is now $d_{\mathrm{peak,city}} = d_u + \Delta d_{\mathrm{peak,city}} = 7.17\,\mathrm{m}$.

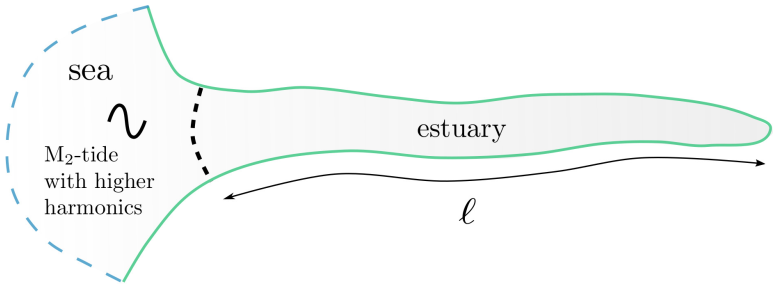
IV.5$^{1.0}$ These waves are the result of disturbances of the water level triggered by the arrival of the wave (opening/closue of gates and weirs, flooding of flood plains). Since these disturbances have much smaller time scales, the effect of inertia can not be ignored - as is the case for the high-water wave itself - while their propagation speed is much faster and also i-directional. At an observation station they appear before the arrival and after the passage of the high-water wave.

Question I (higher harmonics)

An estuary is connected to a tidal sea. The sea is relatively shallow compared to the height of the tidal wave which causes so-called 'shallow-water tides'. These are higher harmonic tidal components having frequencies of $2\omega, 3\omega, \ldots$, etc., in which ω is the principal frequency of the tide.

I.1$^{1.0}$ Mention two distinct generating mechanisms of these higher harmonics and explain why, in general, they become more prominent as the wave height to depth ratio increases.

We consider the response of the estuary to the tide at sea using the harmonic method. For this purpose, the estuary is schematized as a prismatic, semi-closed channel with a length ℓ of 40 km, a conveyance cross section A_c of 20,000 m^2, a width B of 3,500 m, and a linear resistance factor κ of $1.6 \times 10^{-4}\,\mathrm{s}^{-1}$. See the figure below.



I.2$^{1.5}$ For the M_2-tide and its first higher harmonic, calculate the dimensionless parameters that determine the response of the estuary to periodic forcing and use this to characterize the properties of the respective standing waves.

The boundary condition for the surface elevation at the entrance, including the shallow-water tide at sea, is formulated as follows:

$$\zeta_{\mathrm{sea}}(t) = \hat{\zeta}_1 \cos \omega t + \hat{\zeta}_2 \cos 2\omega t$$

in which ω is the frequency of the M$_2$-tide, and $\hat{\zeta}_1$ and $\hat{\zeta}_2$ are the surface elevation amplitudes of the harmonic components which are equal to 0.8 m and 0.3 m, respectively.

I.3$^{1.5}$ Compute the surface elevation amplitude ($\hat{\zeta}$) at the closed end of the estuary of the component having the largest response factor (r).

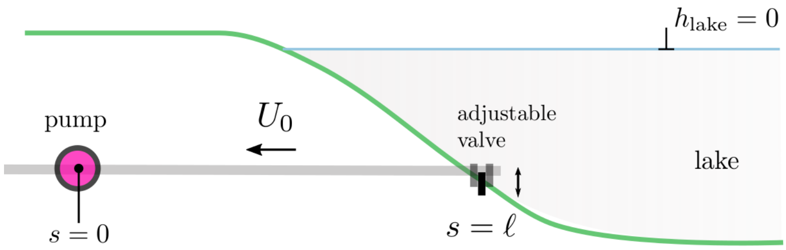
I.4$^{1.0}$ For the same component, compute the amplitude of the discharge at the entrance ($\hat{Q}$).

As with any linear(ized) system, the superposition principle can be applied to obtain a new solution from two (or more) known solutions. Having heard of this, an engineer calculates the tidal wave height at the closed end of the estuary by adding the corresponding wave heights of the respective harmonic components.

I.5$^{1.0}$ Argue whether or not this approach is formally correct.

Question II (cooling water intake)

A power plant takes its cooling water from a large lake by means of a horizontal inlet pipe. The pipe has a pump at one end ($s = 0$) while at the other end ($s = \ell$) an adjustable valve is situated; see the accompanying figure. This question addresses pressure waves in the pipe resulting from manipulations with the pump and/or the valve.



The pipe is made of GRP (E =17 GPa) with a length ℓ of 1,500 m, an inner diameter of 1.2 m, and a pipe-wall thickness of 0.02 m. The bulk modulus of water (K) equals 2.2 GPa.

II.1$^{1.0}$ Calculate the travel time t_ℓ of pressure waves in the pipe.

The discharge relation at the valve is of the following form:

$$h(\ell) = h_{\text{lake}} + \xi \frac{|U(\ell)|(U(\ell)}{2g}$$

in which $h_{\text{lake}} = 0$ is the constant reference water level in the lake and $\xi = 20$ is the head loss coefficient of the valve. The flow resistance in the pipe can be neglected. Initially, a steady state prevails in the pipe with a flow velocity U_0 of -2.0 m/s.

II.2$^{1.0}$ Compute the initial piezometric level h_0 in the pipe (to a precision of one decimal).

At time $t = 0$ the pump is shut down abruptly, the valve is left unaltered. The resulting pressure waves in the pipe are analysed using the method of characteristics using the relations $\mathrm{d}h/\,\mathrm{d}U = \mp c/g$ along the respective characteristic directions $\mathrm{d}s/\,\mathrm{d}t = \pm c$.

II.3$^{1.5}$ For $0 \le t < 4t_\ell$ construct the graphical solution to this problem using the accompanying answer form. The s,t-plane, and the initial - and boundary conditions in the U,h-state diagram should be plotted exactly; the sequence of states in the U,h-state diagram can be plotted schematically.

II.4$^{1.5}$ Compute the value of the maximum piezometric level during the considered time interval and describe the occasion where and when this peak occurs for the first time.

At time $t = 4t_\ell$ the valve is closed fully and abruptly while the pump remains off.

II.5$^{1.0}$ Plot the new boundary condition at $s = \ell$ in the U,h-state diagram and continue the graphical solution until time $t = 5t_\ell$; describe in words how the wave motion in the pipe evolves for $t > 5t_\ell$.

Answer form Questions II.3 and II.5

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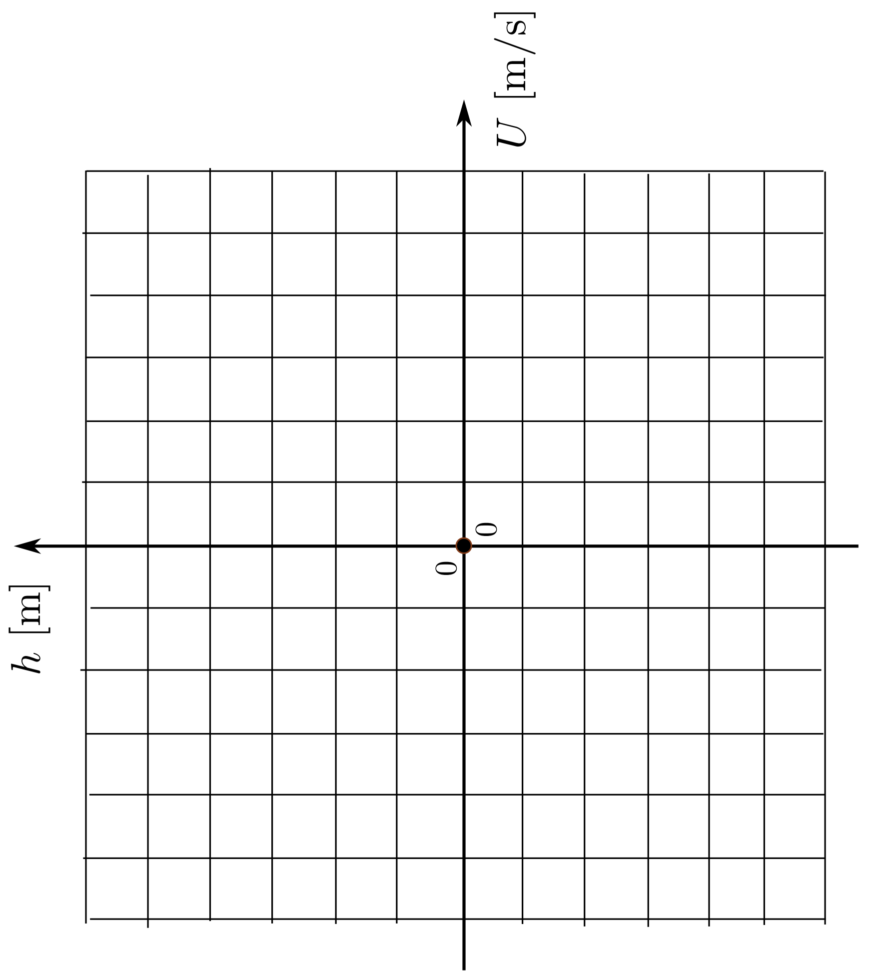
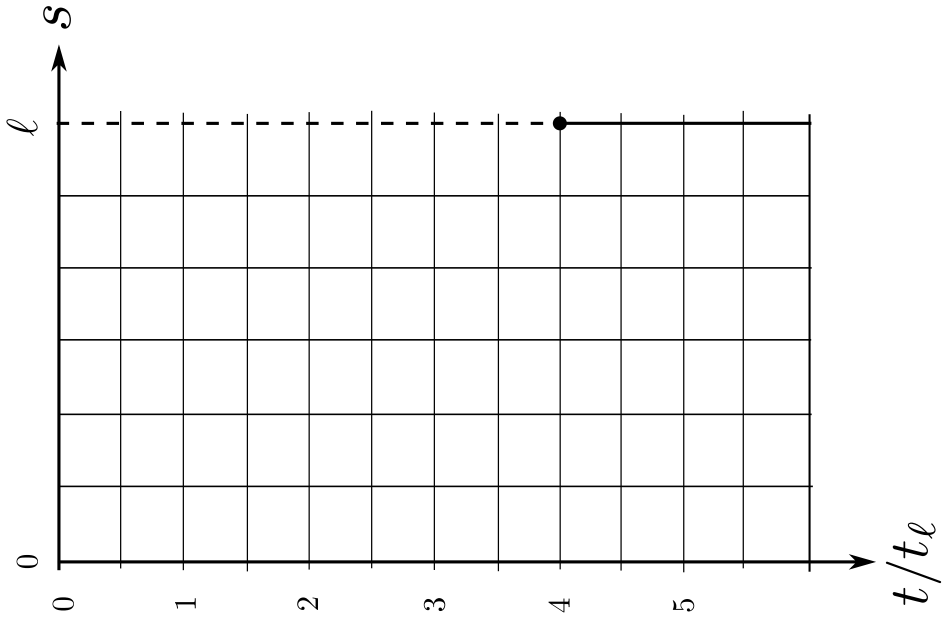
student number:....................................



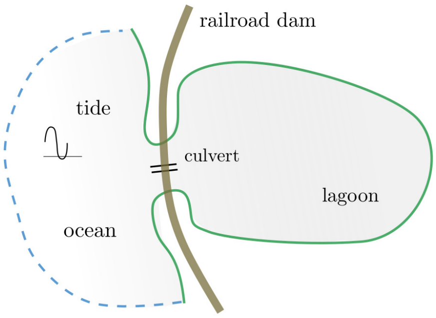
U, h-state diagram



s, t-plane

Question III (lagoon restoration)

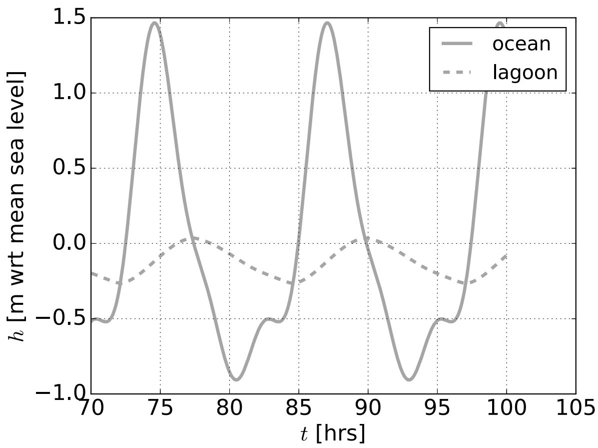
A lagoon is connected to an ocean. A few years ago, the inlet to the lagoon was blocked by a railroad dam while the connection with the ocean was maintained by constructing a culvert through the dam. This all has however severly reduced the tidal discharge through the inlet causing siltation and pollution of the lagoon. See the figure below.



The response of the lagoon and its inlet to the ocean tide is similar to the response of a mass-spring-damper system to a periodic forcing.

III.1[1.0] Explain which parts of the lagoon-inlet system are playing the role(s) of mass, spring and damper, respectively, and mention for each part when the corresponding approximation is valid.

To investigate the present state of the lagoon a field campaign is conducted measuring a.o. the tidal water levels at the ocean and in the lagoon which are plotted in the graph below.



III.2[1.0] Explain how it follows from the above graph that the influence of inertia in the tidal inlet-lagoon system is negligible.

The lagoon has a wet surface area A_b of $12\,\mathrm{km}^2$.

III.3[1.0] Using the above graph, determine the maximum tidal discharge in the culvert.

To improve the tidal flushing of the lagoon, the culvert crossing the dam has to be redesigned. To optimize this design, a linearized model of the lagoon-inlet system is set up using the given surface area A_b for the lagoon, and a conveyance cross section A_c of $50\,\mathrm{m}^2$ for the present culvert. Only the expansion losses at the culvert are taken into account. The ocean tide is schematized as a sinuoidal M_2-tide with a surface elevation amplitude $\hat{\zeta}_s$ of $1.2\,\mathrm{m}$.

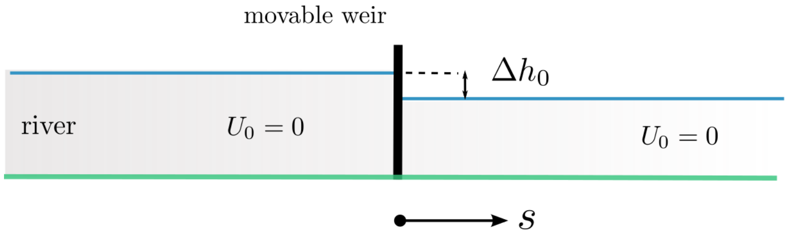
III.4[1.5] Use the linearized model to compute the maximum tidal discharge in the inlet, and conclude whether or not this result is in accordance with the measurements considered before.

In order to restore the lagoon, the tidal discharge in the inlet has to be increased with at least 75%.

III.5[1.5] Use the linearized model to compute the minimum required conveyance cross section A_c of the culvert to meet this design criterion, keeping all other model parameters the same.

Question IV (river weir)

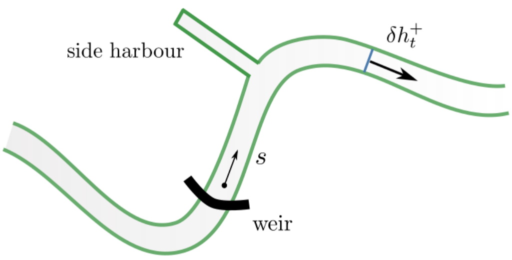
To maintain a sufficiently large water depth in a river a weir has been constructed. The weir is closed if the river discharge is low, thereby setting up the water level in the river reach upstream of the weir. See the figure below, showing a situation where the weir is initially closed.



Anticipating the arrival of a river flood wave, the weir is partially opened leading to a pair of translatory waves travelling away from the weir. The wave travelling in the downstream direction (s positive) has a height δh^+ of 0.25 m. The river has a uniform cross section with a conveyance area A_c of 350 m^2 and width $B = 2\frac{1}{2}B_c = 200$ m. The discharge relation of the weir opening is given by $Q = \mu A\sqrt{2g\Delta h}$, where the effective weir opening μA amounts to 50 m^2.

IV.1$^{1.5}$ Use linear wave theory to compute the corresponding initial water level difference Δh_0 across the weir.

At some distance downstream of the weir (s positive) a side harbour is present involving an abrupt transition of the cross section; see the figure below.



The corresponding reflection coefficient r for the incoming translatory wave (δh^+) amounts to $-\frac{1}{3}$.

IV.2$^{1.0}$ Compute the height and discharge of the transmitted wave that first passes the side harbour, following the course of the river (i.e. δh_t^+ in the figure).

For the approaching flood wave the quasi-uniform approximation holds giving a so-called kinematic wave. An important property of such waves is that for a uniform channel the corresponding characteristics in the s,t-plane are straight lines.

IV.3$^{1.0}$ Explain why the characteristics of a kinematic flood wave are straight lines, and describe at least one important consequence of this regarding the behaviour of the wave.

The maximum water depth at the peak of the flood wave amounts to 8.5 m. Furthermore, the river has a uniform bed slope i_b of 1.4×10^{-4} and a resistance coefficient c_f of 0.005. The cross section is sufficiently wide to assume that the hydraulic radius R equals the water depth d.

IV.4$^{1.5}$ Compute the propagation speed of the peak of the flood wave, and explain why this differs considerably from the propagation speed of the translatory waves issued previously from the weir.

Answers

Question I (higher harmonics)

I.1$^{1.0}$ The higher harmonics result from nonlinear effects. Two types of nonlinearity can be distinguished: 1) products of the dependent variables in the shallow water equations, e.g., the kwadratic resistance term or the advective momentum flux; 2) dependencies of the geometrical parameters on the flow parameters, e.g., the depth or width of a channel depends on the water level. The latter, geometrical, effects become more pronounced with increasing depth - wave height ratio as this determines the sensitivity of the cross sectional parameters to the solution for the waterlevel/depth. Furthermore, the importance of the advective acceleration relative to the local acceleration term is proportional to the Froude-number which also depends on the wave height - depth ratio. Finally, the magnitude of the (nonlinear) resistance factor is proportional to the wave height to depth ratio as, $c_f|U|/R \propto \hat{\zeta}/R$.

I.2$^{1.5}$ The first higher-harmonic has a frequency twice that of the principal component ($\omega = 2\omega_{\mathrm{M}_2} = 2.8 \times 10^{-4}$ rad/s). The two parameters that together determine the response of the estuary to harmonic forcing are the relative resistance $\sigma = \kappa/\omega$ and the basin length to wave length ratio ℓ/L_0, in which the wave length in absense of resistance $L_0 = c_0 T = c_0 2\pi/\omega$ with $c_0 = \sqrt{gA_c/B} = 7.49\,\mathrm{m/s}$. The results for the respective components are summarized in the table below. (Note the factor of two differences recurring in the results collected in the table.)

component	ω (rad/s)	L_0 (km)	σ	ℓ/L_0
M_2-tide	1.4×10^{-4}	336	1.14	0.12
1^{st} higher-harmonic	2.8×10^{-4}	168	0.57	0.24

The basin length to wave length ratio of the higher harmonic is very close to that of the first resonant wave mode (for which $\ell = \frac{1}{4}L_0$) while the relative resistance is moderate; this indicates a large response factor. For the principal component (M_2) the basin length to wave length ratio is well away from the resonant condition while the relative resistance is relatively large, indicating a smaller response factor.

I.3$^{1.5}$ The response factor is defined as $r \equiv \hat{\zeta}_\ell/\hat{\zeta}_0$, where 0 and ℓ refer to the entrance and the closed end, respectively. From the answer to Question II.2 it follows that the higher harmonic (2ω) has the largest response factor in this case. The corresponding amplitude at the closed end is calculated from $\hat{\zeta}_\ell = \hat{\zeta}_0/|\cosh p\ell| = \hat{\zeta}_0/\sqrt{\sinh^2 \mu\ell + \cos^2 k\ell}$. To compute k and μ, first calculate the resistance angle $\delta = \frac{1}{2}\arctan\sigma = 0.26$ rad. This gives $k = k_0/\sqrt{1-\tan^2\delta} = 3.88 \times 10^{-5}\,\mathrm{rad/m}$, using $k_0 = \omega/c_0 = (2.8 \times 10^{-4}\,\mathrm{rad/s})/(7.49\,\mathrm{m/s}) = 3.74 \times 10^{-5}\,\mathrm{rad/m}$, and $\mu = k\tan\delta = 1.03 \times 10^{-5}\,\mathrm{1/m}$. This finally results in $r = 2.36$ and $\hat{\zeta}_\ell = 0.71\,\mathrm{m}$.

I.4$^{1.0}$ The discharge amplitude in the entrance follows from: $\hat{Q}_0 = Bc\cos\delta\,\hat{\zeta}_\ell|\sinh p\ell|$, in which $c = \omega/k = c_0\sqrt{1-\tan^2\delta} = 7.22\,\mathrm{m/s}$, and $|\sinh p\ell| = \sqrt{\sinh^2\mu\ell + \sin^2 k\ell}$. Substitution of δ, k, μ and $\hat{\zeta}_\ell$ gives $\hat{Q}_0 = 18{,}750\,\mathrm{m^3/s}$.

I.5$^{1.0}$ The solution of the (linear) harmonic wave equation yields a complex amplitude (i.e. $\tilde{\zeta}, \tilde{Q}$). The superposition principle can therefore be applied to the complex amplitudes but not to their absolute values (i.e. the amplitudes) since $|\tilde{\zeta}_1 + \tilde{\zeta}_2| \neq |\tilde{\zeta}_1| + |\tilde{\zeta}_2|$. For two or more harmonic components, the total wave height - or surface elevation amplitude - not only depends on the individual wave heights of the respective components but also on their mutual phase differences.

Question II (cooling water intake)

II.1$^{1.0}$ The travel time $t_\ell = \ell/c$. The wave speed $c = 1\sqrt{\rho/K + \rho D/\delta E} = 501\,\mathrm{m/s}$, giving a travel time $t_\ell = 3.0\,\mathrm{s}$.

II.2$^{1.0}$ Since there are no wall friction losses in the pipe and the flow velocity is steady and uniform, the piezometric level in the pipe is uniform and equal to that at the inflow end. From the corresponding boundary condition at $s = \ell$ it follows that $h_0 = h_\ell = h_{\text{lake}} - \xi U_0^2/2g =$ -4.1 m.

II.3$^{1.5}$ See the accompanying Figure 1.

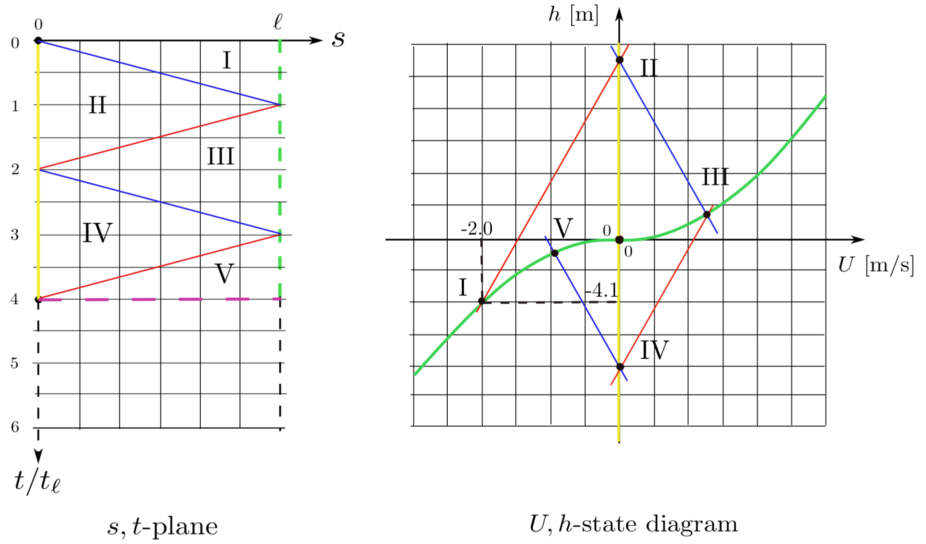


Figure 1: s, t-plane and U, h-state diagram Question II.3

II.4$^{1.0}$ For $0 \leq t < 4t_\ell$, the largest piezomeric level occurs for state II. This state is associated with the positive pressure wave that is issued from the pump when it is closed, which travels into the positive s-direction. Using the characteristic relation along corresponding negative characteristic the value of h_{II} follows from: $h_{II} = h_I + (c/g)(U_{II} - U_I)$. Since $h_I = h_0$ and $U_{II} - U_I = 0 - U_0 = 2.0\,\mathrm{m/s}$, it follows that $h_{II} = h_0 - U_0\, c/g =$ -4.08 m - (-2.0 m/s) $\times$ (501 m/s) / (9.81 m/s^2)=98.1 m.

II.5$^{1.5}$ The boundary conditions at the pump and the valve ($U = 0$) coincide in the state-diagram. The solution is that of a periodic motion whith a standing wave oscillation in the pipe (with a wave length of 2ℓ and a period of $2t_\ell$). See the accompanying Figure 2.

Question III (lagoon restoration)

III.1$^{1.0}$ The lagoon-inlet system has two components:

1. A conveying component (inlet): the water mass flowing in the entrance corresponds to the mass in a mass-spring-damper system as it represents inertia; the flow resistance occuring at the entrance (expansion losses and wall resistance) are damping forces and can be associated with the damper in a mass-spring-damper system.

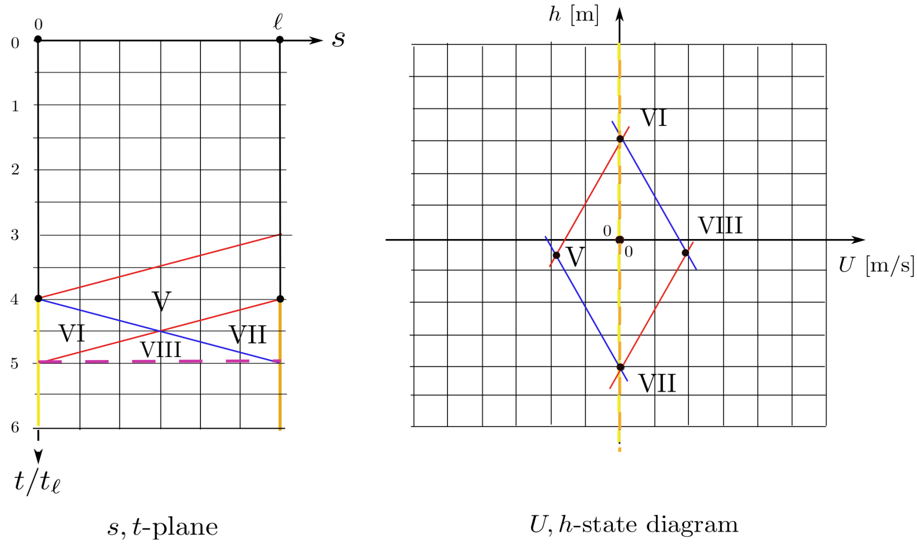


Figure 2: s,t-plane and U,h-state diagram Question II.5

2. A storing component (lagoon): the surface level in the lagoon corresponds to the spring force in a mass-spring-damper system; for an unforced situation (no tide), a deviation (displacement) of the surface level from the mean sea level will accelerate the water mass in the entrance.

The above schematization of the lagoon-inlet is valid if the discharge in the entrance and the water level in the lagoon are uniform, depending on time only. For the conveying component (inlet) this will be the case if its length is small relative to the (tidal) wave length, and if its storage area is small relative to that of the lagoon (rigid-column approximation). For the storing component (lagoon) the horizontal dimensions must also be small relative to the wave length, and the friction slope due to the flow within the lagoon must be negligible; only weak currents due to filling/emptying (small basin approximation).

III.2$^{1.5}$ The graph shows that the water level in the lagoon has a maximum or minimum if it is equal to the simultaneous water level at the ocean. This implies that the discharge at the entrance becomes zero (because $\mathrm{d}h_b/\mathrm{d}t = 0$) as soon as the water level difference between the lagoon and the ocean is zero. The discharge is then a function of the water level difference between lagoon and ocean only, which is possible only if the inertia term in the momentum equation can be ignored (see Eq. 6.11 in the textbook).

III.3$^{1.5}$ The continuity equation for the lagoon states that the discharge in the inlet is proportional to the rate with which the water level in the lagoon changes. The later is indicated by the slope of the water level graph. The maximum value of this slope occurs between ca. 85-90 hrs and is estimated as $\mathrm{d}h_b/\mathrm{d}t|_{\max} \approx (0.25\text{ m})/(3\text{ hrs}) = 2.3\times10^{-5}\,\mathrm{m/s}$. The corresponding estimated maximum discharge in the inlet is $Q_{\max} = A_b\ \mathrm{d}h_b/\mathrm{d}t|_{\max} \approx 280\,\mathrm{m^3/s}$ (towards the lagoon).

III.4$^{1.0}$ The discharge amplitude in the culvert is given by $\hat{Q} = \omega A_b \hat{h}_b$, in which $\hat{h}_b = r\hat{h}_s$. The response factor r is calculated from (system with resistance and storage): $r = (1/\sqrt{2}\Gamma)\sqrt{\sqrt{4\Gamma^2+1}-1}$ where $\Gamma = (8/3\pi)\chi(\omega A_b/A_c)^2\hat{h}_s/g$ (see formula sheet). Using the given quantities and $\chi = \frac{1}{2}$ (expansion lossen only) we obtain $\Gamma = 58.6$ and subsequently $r = 0.13$. This gives a surface level amplitude in the lagoon $\hat{h}_b = 0.16\,\mathrm{m}$, and finally a discharge amplitude $\hat{Q} = 262\,\mathrm{m^3/s}$. The latter is reasonably close to the maximum discharge estimated from the graph (considering the linearization of the model and the presence of higher harmonics in the original tidal signal).

III.5$^{1.0}$ If the discharge has to increase with 75%, the response factor r has to increase with 75% to a value of $r = 0.23$. The corresponding value of Γ giving this response factor can be calculated using $r = 1/\sqrt{1+(\Gamma r)^2}$. Inverting this gives $\Gamma = \sqrt{(1-r^2)/r^4} = 18.8$. Since Γ is inversely proportional to the square of the conveyance area we have $A_{c,\text{new}}/A_{c,\text{old}} = \sqrt{\Gamma_{\text{old}}/\Gamma_{\text{new}}}$ from which it follows that $A_{c,\text{new}} = 88\,\mathrm{m^2}$.

Question IV (river weir)

IV.1[1.5] The discharge associated with the translatory wave (which equals the discharge through the weir opening) is given by $Q = Bc\,\delta h^+ = 207.2\,\mathrm{m}^3/\mathrm{s}$, using a wave speed $c = \sqrt{gA_c/B} = 4.14\,\mathrm{m/s}$. From the discharge elevation for the weir, the corresponding water level difference across the weir equals $\Delta h = (Q/\mu A)^2/2g = 0.88\,\mathrm{m}$. Considering the heights of the right - and left travelling waves, $\delta h^+ = 0.25\,\mathrm{m}$ and $\delta h^- = -\delta h^+ =$ -0.25 m, respectively, it folows that the initial water level difference across the weir equals $\Delta h_0 = \Delta h + \delta h^+ - \delta h^- = 1.38\,\mathrm{m}$.

IV.2[1.0] The water level elevation at the side harbour shortly after reflection amounts to $\delta h = \delta h_i^+ + r\delta h_i^+ = \frac{2}{3}\delta h_i^+$ (where $\delta h_i^+ = \delta h^+$ from the previous question). This is also the height of the transmitted wave (water level continuous at transition); therefore $\delta h_t^+ = \frac{2}{3}\delta h_i^+ = 0.17\,\mathrm{m}$. The corresponding discharge is $Q_t^+ = Bc\,\delta h_t^+ = 138\,\mathrm{m}^3/\mathrm{s}$.

IV.3[1.0] The solution of the kinematic wave equation states that along characteristics $\mathrm{d}s/\,\mathrm{d}t = c_{HW}$ in the s,t-plane the cross section A is constant. For a uniform channel, the wave speed c_{HW} is a function of the solution A and the (constant) cross-sectional parameters i_b, c_f, B and B_c. It follows that for a constant value of A along each characteristic, also the characteristic speed c_{HW} is constant along each characteristic. Hence, the characteristics are straight lines. The important consequence of this is that the wave height does not change during propagation. The wave may deform however because points with different depths travel with different speeds.

IV.4[1.5] The high-water wave speed is given by $c_{HW} = \frac{3}{2}(B_c/B)U_u$, in which U_u is the uniform flow velocity corresponding to the local water depth. Using $U_u = \sqrt{gRi_b/c_f} = 1.53\,\mathrm{m/s}$, the wave speed equals $c_{HW} = 0.92\,\mathrm{m/s}$. This is considerably smaller than the wave speed c of the (linear) translatory waves because inertia does not play a role for the flood wave, which is dominated by resistance. The acceleration terms are thus ignorted leading to a (much) smaller wave speed.

Faculty Civil Engineering and GeoSciences

Exam	CTB3350 (BSc) CIE3310-09 (MSc)
	Open Channel Flow
Total number of pages	6 pages
Date and time	June 28, 2017 at 9:00 - 12:00 hours
Responsible lecturer	R. J. Labeur

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of questions: 4 (of which 4 open questions and 0 multiple choice questions)

Max. number of points to be granted: 22

☐ all questions have equal weight
☑ questions differ in weight (the weight is mentioned per question, or is given in an overview)

Use of tools and sources of information (to be filled in by course examiner)

Not allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with pencil.
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ books ☐ notes ☐ dictionaries ☐ syllabus
☑ formula sheets (see also below under 'additional information') ☑ calculators ☐ computers
☐ ...

Additional information (if necessary to be filled in by the examiner)

The number of points that can be obtained for each subquestion is shown next to the respective question number. When grading your exam, special attention will be given to a correct application of the underlying theory and a systematic approach to the problem. Provide your answers with a short description of the chosen solution strategy. If you need to make assumptions or simplifications briefly explain why it is allowed to do so. Where a previous answer is needed in a subsequent question, but which you were not able to compute, assume a plausible numerical value. Numerical answers should contain the correct dimensional units, at least in the final answer but preferably also in the intermediate steps. Provide your answers on a

Exam graded by: July 18, 2017 (to be filled in by course examiner)

(the marking period is 15 working days at most)

!

Every suspicion of fraud is reported to the Board of Examiners.

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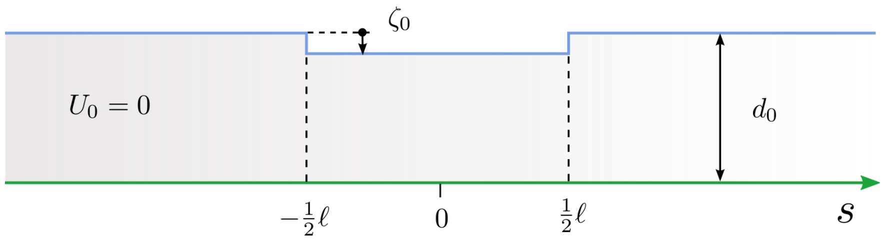
Question I (Meteo-tsunami)

On May 29, 2017, a so-called meteo-tsunami occurred along the North-Sea coast of The Netherlands causing flooding of properties along the beach. Phenomena like this are caused by strong, rapid fluctuations of the atmospheric pressure leading to low initial perturbations of the sea surface that propagate towards the coast as long waves.

I.1$^{1.0}$ Explain how these initially low waves can result in waves with considerable height once they enter shallower near-shore regions or harbours.

For preliminary analyses of the meteo-tsunami event, the corresponding initial displacement of the sea surface ζ_0 is schematized as an instantaneous, uniform depression extending over a horizontal distance ℓ in water having a constant undisturbed depth d_0 and zero initial velocity. For simplicity we consider the situation per unit width. See the figure below showing the initial condition at time $t = 0$ of the corresponding wave problem.



The undisturbed depth d_0 amounts to 40 m, the initial surface displacement ζ_0 equals -0.20 m, and the wavelength ℓ equals 40 km.

I.2$^{1.0}$ Verify by means of numerical estimates that the resulting wave can be considered a linear shallow water wave.

I.3$^{1.5}$ Make two separate plots of the surface elevation $\zeta(s)$ and the flow velocity $U(s)$, respectively, at time $t = 500$ s; provide your figures with the relevant numerical values of the flow states (U and ζ) and of the coordinates defining the spatial intervals in which the different states occur.

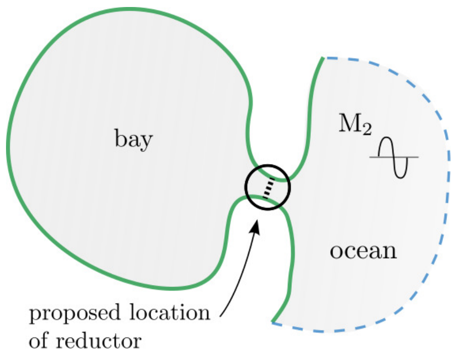
Towards the coast the undisturbed water depth gradually decreases uniformly in the propagation direction of the waves. In such cases Green's law holds for translatory waves, stating that the wave height is inversely proportional to the fourth root of the depth, or equivalently, $\zeta/\zeta_0 = (d_0/d)^{1/4}$.

I.4$^{1.0}$ Explain which principle Green used to derive this law.

I.5$^{1.5}$ Use Green's law to compute the wave height ζ and corresponding flow velocity U at a water depth of 5 m, and comment whether or not the linear wave assumption is still applicable at this location for this particular wave.

Question II (Reductor)

A tidal bay is connected to the ocean by a short and narrow gap. In the ocean an M_2 tide prevails. During extreme spring tides the areas surrounding the bay are flooded. As a possible flood protection measure it is proposed to construct a so-called reductor in the inlet of the bay. This is a semi-open barrier which will locally increase the flow resistance, thereby reducing the tidal amplitude in the bay. See the figure below for an overview of the proposed situation.



The tidal basin has an undisturbed depth d_0 of 6 m, a surface area A_b of 80 km^2, and horizontal dimensions of approximately 9 km × 9 km.

II.1[1.0] Compute the range of forcing frequencies ω for which the bay will behave as a small basin, and verify that the M_2 tide satisfies this criterion.

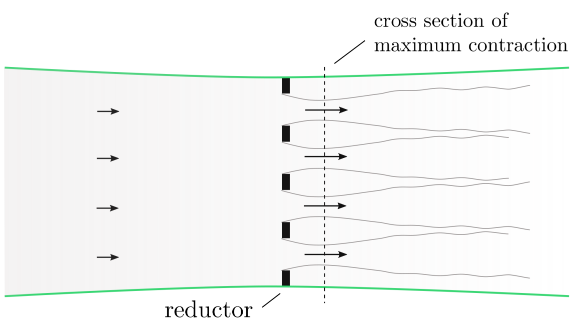
In the original situation (without reductor) the inlet of the bay has a conveyance cross-section A_c of 2,500 m^2. During spring tide the surface elevation amplitude at the ocean $\hat{\zeta}_s$ amounts to 2.50 m.

II.2[1.0] Compute the corresponding surface elevation amplitude in the bay. (Use a value of 1.85 m in the remainder of this Question if you are not able to obtain an answer.)

To prevent flooding, the surface elevation amplitude in the bay during spring tide has to be reduced with 30% compared to the corresponding amplitude in the existing situation. This has to be accomplished by the proposed reductor which will increase the resistance parameter χ of the inlet.

II.3[1.5] Compute the required value of the resistance parameter χ of the inlet in order to prevent flooding of the areas around the bay during spring tide. (Use a value of 1.25 in the remainder of this Question if you are not able to obtain an answer.)

The resistance of the reductor is caused by contraction and subsequent expansion of the flow passing the construction, see the figure below.

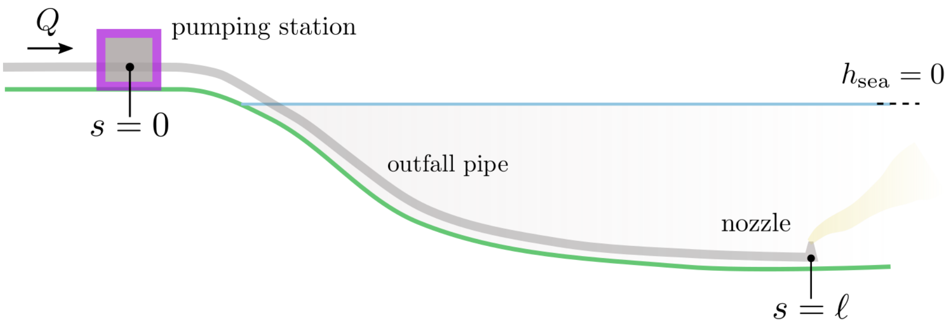


The expression for the corresponding resistance parameter reads $\chi = \frac{1}{2}\mu^{-2}$, in which μ is the total contraction coefficient, i.e. the ratio of the conveying part of the cross section at maximum contraction and the total conveying cross section A_c of the bay inlet.

II.4[1.5] Compute the tidal velocity amplitude $\hat{U}$ within the cross section of maximum contraction.

Question III (Outfall pipe)

A wastewater treatment plant releases effluent into the open sea by means of an outfall pipe with a constant diameter. The shore based pipe end ($s = 0$) features a pumping station. The other, submerged, end of the pipe ($s = \ell$) has a so-called diffusor, a narrow nozzle through which the water disperses with a high flow velocity − to enhance mixing − into the environment. See the figure below for an overview of the situation.



In the past, several such outfall pipes have collapsed due to the formation of vapour bubbles in the pipe caused by extemely low fluid pressures.

III.1[1.0] Explain, in the context of pressurized pipe flow, how these extreme pressures in the pipe can result from manipulation of the controls of the pipe (i.e. pumps, valves, etc).

We study the occurrence of such pressure waves in the outfall pipe using the method of characteristics. The characteristic equations read $\mathrm{d}h/\,\mathrm{d}U = \mp c/g$ along $\mathrm{d}s/\mathrm{d}t = \pm c$, in which c is the (constant) propagation speed of pressure waves in the pipe. The initial flow velocity U_0 in the pipe equals 2.0 m/s. Following a full and abrupt closure of the pumping station, a negative pressure wave is issued from $s = 0$ with a corresponding piezometric level increment Δh^+ of −85 m.

III.2[1.0] Compute the propagation speed of this wave.

The boundary condition at $s = \ell$ (nozzle) is given by $h(\ell) = h_{\text{sea}} + \xi |U(\ell)| U(\ell)/2g$, in which the discharge coefficient ξ of the nozzle equals 50 and the sea level h_{sea} equals zero. All energy losses in the pipe other than the exit loss at the nozzle can be ignored.

III.3[1.0] Calculate the initial piezometric level h_0 in the pipe.

At $t = 0$ the pump is fully closed leading to the boundary condition $U(0) = 0$ for $t > 0$.

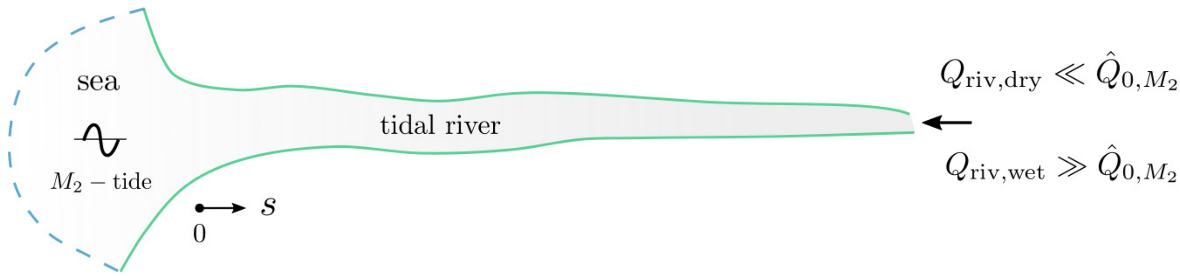
III.4[1.5] Plot the s, t diagram and the corresponding U, h state diagram for $0 < s < \ell$ and $0 < t < 5\,t_\ell$, where $t_\ell = \ell/c$ is the travel time of pressure waves in the pipe. Use the accompanying answer form; plot the initial - and boundary conditions accurately using the respective numerical values; the slopes of the characteristic relations may be plotted schematically.

The treatment plant will be upgraded leading to an increase of the wastewater discharge Q. Preferably, the existing outfall pipe is used to accomodate the increased discharge. However, the minimum piezometric level $h_{\min}$ in the pipe after an emergency closure of the pump must remain higher than −100 m (with respect to the given reference level) to warrant the structural integrity of the pipe.

III.5[1.5] Explain which intersection point in the U, h diagram corresponds to this situation and compute the maximum allowable initial velocity $U_{0,\max}$ for which the safety criterion is met.

Question IV (Tidal river)

A very long tidal river is connected to a sea in which an M_2 tide prevails. During most of the year the river discharge is negligible and the flow state in the river is dominated by the tide. During the wet season however, the river discharge increases dramatically and completely suppresses the propagation of tidal waves into the river. See the below figure.



We will first consider the dry season with a low river discharge. In this situation the tidal wave in our river is a *progressive* wave. Generally, a progressive tidal wave occurs if the product of the damping parameter μ of the tidal wave and the length ℓ of the river is large.

IV.1$^{1.0}$ Explain why for rivers with $\mu\ell \gtrsim 3$ the tidal wave in the river is a progressive wave rather than a standing wave.

The width of such rivers often decreases gradually in inland direction but for preliminary calculations we will ignore such variations, and simplify the river using a uniform cross section having a width $B = 2.5\,B_c = 5{,}000\,\text{m}$ and a conveyance cross-section $A_c = 30{,}000\,\text{m}^2$. The tidal wave is modelled using the harmonic method with a corresponding linear resistance factor κ of $4.0 \times 10^{-4}\,\text{s}^{-1}$.

IV.2$^{1.0}$ What is the minimum length ℓ of our tidal river, considering that the tidal wave is progressive and using the previously given criterion?

The tidal surface level elevation at the entrance of the river ($s = 0$) amounts to $\hat{\zeta}_0 = 0.85\,\text{m}$.

IV.3$^{1.5}$ Compute the tidal surface level elevation amplitude $\hat{\zeta}$ and the corresponding tidal discharge amplitude $\hat{Q}$ at a distance of half a wave length from the mouth, that is at $s = \frac{1}{2}L$ where L is the tidal wave length in the river.

During the wet season a high-water wave occurs in the river with a corresponding peak discharge $Q_{\text{riv,wet}}$ of $120{,}000\,\text{m}^3/\text{s}$ which by far exceeds the tidal discharge, hence the neglect of the tidal wave in this situation. This high-water wave is a so-called kinematic wave. An imaginary observer follows the peak of the high-water wave while measuring the water depth.

Additional data: bed slope of the river $i_b = 2 \times 10^{-4}$, resistance coefficient $c_f = 0.005$; furthermore, use $R = d$ and $A_c = B_c d$.

IV.4$^{1.5}$ Compute the speed of the observer with respect to the river bed, and explain why the water depth measured by the observer does not vary in time.

Answers

Question I (Meteo-tsunami)

I.1 When (long) waves approach the shore, the magnitude of the surface level elevation generally increases due to the decreasing water depth. The shallower depth slows down the wave speed. For an abrupt depth change this leads to positive reflection with a transmitted wave height larger than the incoming wave height. For a gradual depth change a similar effect occurs (think of it as an infinite number of small abrupt depth changes). In addition, the penetration of these waves into harbour basins comes with resonance effects if the period of the wave is approximately equal to one of the eigenperiods of the harbour, also giving an amplification of the surface level amplitude.

I.2 To allow for a linearization of the problem, the magnitude of the surface level elevation should be much smaller than the water depth. In this case we have $|\zeta|/d_0 = 0.20\,\mathrm{m}/40 \times 10^3\,\mathrm{m} = 0.5 \times 10^{-5}$ which clearly satisfies the criterion. For a wave to be a shallow water wave, the horizontal length scale of the disturbance should be much larger than the water depth, or $\ell \gg d_0$. In this case we have $\ell/d_0 = 40 \times 10^3\,\mathrm{m}/40\,\mathrm{m} = 10^3$ which satisfies the criterion.

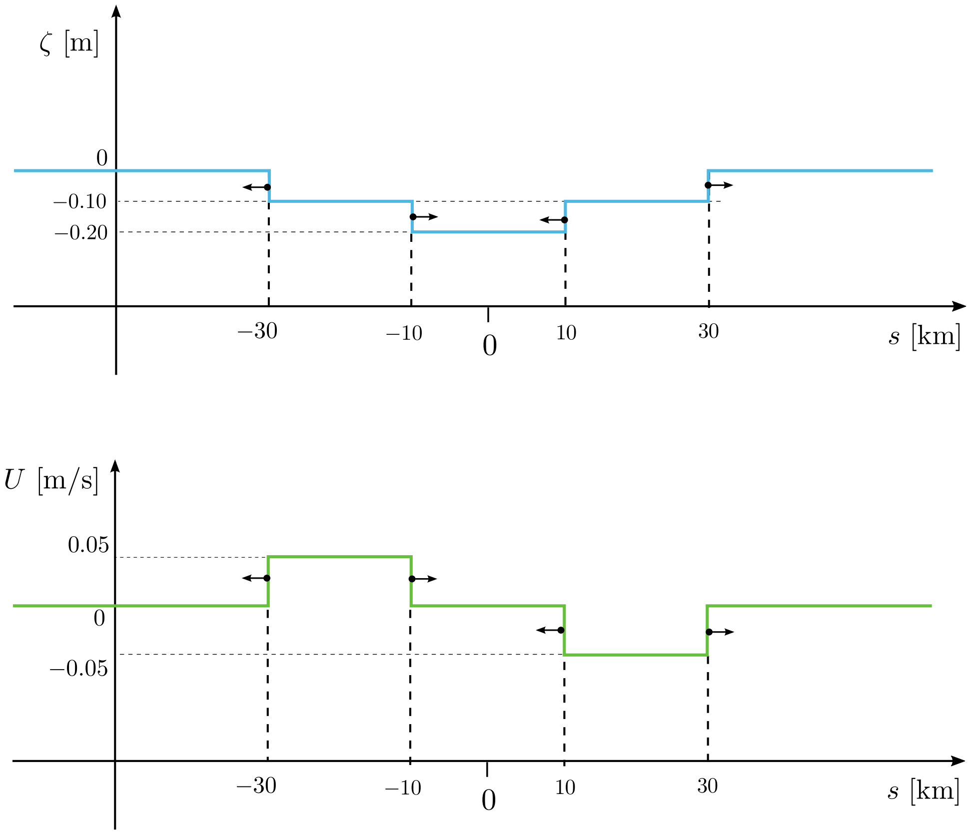
I.3 Using linear wave theory, the initial disturbance splits into two wave components, a component travelling to the right with corresponding initial surface elevation $\zeta_0^+ = (\zeta_0 + U_0 c/g)/2$, and a component travelling to the left having an initial surface elevation $\zeta_0^- = (\zeta_0 - U_0 c/g)/2$. In this case the initial velocity equals zero, leading to $\zeta_0^+ = \zeta_0^- = \zeta_0/2 = -0.10\,\mathrm{m}$. The corresponding initial flow velocities are, respectively, $\delta U_0^+ = (c/g)\zeta_0^+ = -0.05\,\mathrm{m/s}$ and $\delta U_0^- = -(c/g)\zeta_0^- = 0.05\,\mathrm{m/s}$, using a wave speed $c = \sqrt{gd_0} = 19.8\,\mathrm{m/s}$. The solution at time $t = 500\,\mathrm{s}$ is obtained by shifting these components to the right and left, respectively, over the travelled distances $\Delta s = \pm c\Delta t = \pm 9{,}900\,\mathrm{m}$ (say 10 km), after which they are superimposed. The resulting solutions for the surface elevation and the flow velocity are depicted in the accompanying figure.

I.4 Green derived this law by stating that, in absence of resistance, the total energy E in a wave must remain constant. Considering a situation per unit width, this energy is proportional to the square of the surface elevation times the length of the wave: $E \propto \zeta^2 \ell$. Since the length ℓ of a disturbance is proportional to the local wave speed (the wave period or duration of a wave is constant) it follows that $E \propto c\zeta^2$. For the energy to remain constant during propagation into shallower water it follows that $d^{1/4}\zeta$ is constant along the path of the wave leading to the given expression.

I.5 Apply Green's law to the wave component travelling towards the coast, component ζ^+, say; using $d_0 = 40\,\mathrm{m}$ and $\zeta_0^+ = -0.10\,\mathrm{m}$ this gives $\zeta^+ = -0.17\,\mathrm{m}$ for $d = 5\,\mathrm{m}$. The corresponding flow velocity equals $\delta U^+ = (g/c)\zeta^+ = -0.24\,\mathrm{m/s}$, using the local wave speed $c = \sqrt{gd} = 7.0\,\mathrm{m/s}$. The local wave height magnitude to water depth ratio equals $|\zeta^+|/d = 0.03 \ll 1$ so the linear wave approximation still holds at this location.

Question II (Reductor)

II.1 The small basin approximation is valid if the bay a one or more openings through which it is filled and emptied (no flow through the basin) and if the horizontal dimensions of the bay are small with respect to the local wave length. For the latter the following criterion is commonly used: $\ell/L < 1/20$, where ℓ is the horizontal dimension of the basin and L is the local wave length. Using $L = cT = c2\pi/\omega$,



Solution of Question I.3; arrows indicate propagation direction of the respective wave fronts

it follows that $\ell\omega/c < \pi/10$, or $\omega < (\pi/10)c/\ell$. Using $\ell = 9{,}000\,\mathrm{m}$ and $c = \sqrt{gd} = 7.67\,\mathrm{m/s}$ it follows that $0 < \omega < 2.68 \times 10^{-4}$ rad/s. The M_2 tide ($\omega = 1.4 \times 10^{-4}$ rad/s) lies within the allowable range.

II.2 Use the approach for a tidal basin with storage and resistance. Compute first the resistance parameter $\Gamma = \frac{8}{3\pi}\chi(\omega A_b/A_c)^2\hat{\zeta}_s/g$. Using $\chi = 1/2$ (small gap) and inserting the values of the other parameters gives $\Gamma = 2.17$. The amplitude ratio follows from $r = (1/\sqrt{2}\Gamma)\sqrt{\sqrt{1+4\Gamma^2}-1} = 0.61$. The surface level amplitude in the bay becomes $\hat{\zeta}_b = r\hat{\zeta}_s = 1.51\,\mathrm{m}$.

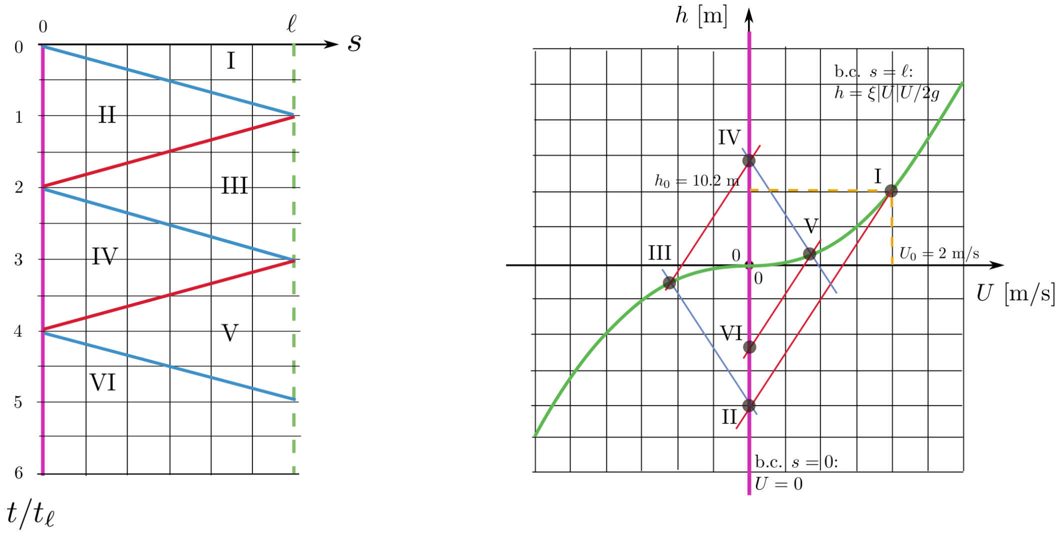
II.3 The amplitude ratio r must be reduced with 30% giving $r = 0.7 \times 0.61 = 0.42$. For the amplitude ratio it holds that $r = 1/\sqrt{1+(\omega\tau)^2}$ which can also be written as $r = 1/\sqrt{1+(\Gamma r)^2}$, since $\omega\tau = \Gamma r$. Inverting this expression gives $\Gamma = (1-r^2)/r^4$ which for the required value of r leads to $\Gamma = 5.04$. The increase of Γ is entirely due to the resistance parameter χ which must therefore increase from $\chi = 1/2$ to $\chi = (1/2) \times (5.04/2.17) = 1.16$.

II.4 First compute the discharge amplitude in the bay entrance: $\hat{Q} = \omega A_b\hat{\zeta}_b = 11{,}867\,\mathrm{m^3/s}$. The corresponding flow velocity amplitude at the cross section of maximum contraction equals $\hat{U} = \hat{Q}/(\mu A_c)$. Using the given expression for the contraction coefficient gives $\mu = 1/\sqrt{2\chi} = 0.66$, and $\hat{U} = 7.23\,\mathrm{m/s}$.

Question III (Outfall pipe)

III.1 Due to the high stiffness of the pipewall and the near-incompressibility of the fluid in the pipe (water), mass storage in a cross section ($\partial\rho A/\partial t$) resulting from gradients in the mass flow ($\partial\rho Q/\partial s$) requires very large pressure changes. Even small velocity changes, resulting for instance from manipulating the flow controls (pumps, valves) may thus cause very large pressure variations.

III.2 For a pressure wave propagating in the positive $s-$direction we have $\Delta h^+ = (c/g)\Delta U^+$. Fol-



s, t-plane $\quad\quad$ U, h-state diagram

Solution of Question III.4; slopes of characteristic relations in the U, h state diagram not to scale

lowing an abrupt and full closure of the pump, a pressure wave with $\Delta U^+ = -U_0 = -\,2.0\,\text{m/s}$ is issued from $s = 0$. Using the above relation for this pressure wave and the given value of $\Delta h^+ = -85\,\text{m}$ it follows that $c = g\Delta h^+/\Delta U^+ = 417\,\text{m/s}$.

III.3 Without resistance in the pipe, the piezometric level in the pipe is constant for steady uniform flow. Therefore, $h_0 = h(\ell)$. The latter follows from the initial flow velocity and the given discharge relation of the nozzle, resulting in $h_0 = h_{\text{sea}}(= 0) + \xi U_0^2/2g = 10.2\,\text{m}$.

III.4 See the accompanying figure.

III.5 The initial flow velocity U_0 influences both the initial piezometric level in the pipe ($h_0 = \xi U_0^2/2g$) as well as the pressure variation of the wave issued from the pump after closure ($\Delta h^+ = -(c/g)U_0$). From the state diagram can be seen that the critical condition for the minimum piezometric level occurs in state II, with a corresponding piezometric level $h_{II} = h_0 - (c/g)U_0 = \xi U_0^2/2g - (c/g)U_0$. For this value to be larger than $-100\,\text{m}$ we must require that $\xi U_0^2/2g - (c/g)U_0 > -100$ m. Solving this equation for U_0 gives a maximum initial velocity of $2.83\,\text{m/s}$.

Question IV (Tidal river)

IV.1 For a progressive tidal wave the amplitudes of the surface level elevation and the discharge decrease in proportion to $\exp(-\mu s)$. For a river of length ℓ, the surface elevation amplitude of the incoming tidal wave at the closed end will be $\exp(-\mu\ell)$ times the amplitude of the surface elevation of the wave entering the river at $s = 0$. If $\mu\ell$ is large, larger than 3 say, the height of the incoming wave at $s = \ell$ is negligible and so is the height of the reflected wave reflected at the closed end. A standing wave will therefore virtually not occur resulting in a progressive tidal wave in the entire river.

IV.2 For a progressive wave $\ell \gtrsim 3/\mu$. The damping parameter is computed as follows. For $\kappa = 4 \times 10^{-4}\ \text{s}^{-1}$, $\sigma = \kappa/\omega = 2.86$, and $\delta = \frac{1}{2}\arctan\sigma = 0.61\,\text{rad}$. This gives for the wave number $k = k_0/\sqrt{1 - \tan^2\delta} = 2.59 \times 10^{-5}$ rad/s, using $k_0 = \omega/c_0 = 1.82 \times 10^{-5}$ rad/m and $c_0 = \sqrt{gA_c/B} = 7.67\,\text{m/s}$. We then obtain $\mu = k\tan\delta = 1.84 \times 10^{-5}\ \text{m}^{-1}$. Using the criterion for a progressive tidal wave, we finally obtain $\ell \gtrsim 160\,\text{km}$.

IV.3 For a progressive wave the surface elevation amplitude is given by $\hat{\zeta}(s) = \hat{\zeta}_0 \exp(-\mu s)$. Inserting $s = \frac{1}{2}L = \pi/k$ this gives $\hat{\zeta}(L/2) = \hat{\zeta}_0 \exp(-\pi\mu/k) = \hat{\zeta}_0 \exp(-\pi\tan\delta)$. Using $\delta = 0.61$ rad and $\hat{\zeta}_0 =$

0.85 m this gives $\hat{\zeta}(L/2) = 0.09\,\mathrm{m}$. The correponding discharge amplitude follows from (progressive wave) $\hat{Q}(L/2) = Bc\cos\delta\,\hat{\zeta}(L/2) = 2017\,\mathrm{m^3/s}$, using $c = \omega/k = 5.41\,\mathrm{m/s}$.

IV.4 The observer moves with the propagation speed of the top of the kinematic wave given by $c_{HW} = \frac{3}{2}(B_c/B)\,U$ in which U is the flow velocity at the top of the wave. To compute the latter from the given peak discharge we use the fact that for a kinematic wave the discharge depends on the local water depth and the local cross-sectional parameters, as follows: $Q = A_c\sqrt{gRi_b/c_f}$. Using $A_c = B_c d$ and $R = d$ this can be written as $Q = B_c d\sqrt{gdi_b/c_f}$ which gives an expression for the depth $d^3 = c_f q^2/gi_b$. This results in a depth at the wave top of $d = 20.93\,\mathrm{m}$. The flow velocity at the wave top equals $U = Q/B_c d = 2.87\,\mathrm{m/s}$, and finally the wave speed $c_{HW} = 1.72\,\mathrm{m/s}$.

The kinematic wave equation reads $\partial A/\partial t + c_{HW}\partial A/\partial s = 0$ stating that the cross section does not change in a frame of reference moving with the wave speed c_{HW}. For a uniform cross section this implies a constant depth for an observer moving along with the wave speed.

Faculty Civil Engineering and GeoSciences

TU Delft

Exam	CTB3350 (BSc) CIE3310-09 (MSc)
	Open Channel Flow
Total number of pages	6 pages
Date and time	June 28, 2017 at 9:00 - 12:00 hours
Responsible lecturer	R. J. Labeur

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of questions: 4 (of which 4 open questions and 0 multiple choice questions)

Max. number of points to be granted: 22

☐ all questions have equal weight
☑ questions differ in weight (the weight is mentioned per question, or is given in an overview)

Use of tools and sources of information (to be filled in by course examiner)

<u>Not</u> allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with <u>pencil</u>.
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ books ☐ notes ☐ dictionaries ☐ syllabus
☑ formula sheets (see also below under 'additional information') ☑ calculators ☐ computers
☐ ...

Additional information (if necessary to be filled in by the examiner)

The number of points that can be obtained for each subquestion is shown next to the respective question number. When grading your exam, special attention will be given to a correct application of the underlying theory and a systematic approach to the problem. Provide your answers with a short description of the chosen solution strategy. If you need to make assumptions or simplifications briefly explain why it is allowed to do so. Where a previous answer is needed in a subsequent question, but which you were not able to compute, assume a plausible numerical value. Numerical answers should contain the correct dimensional units, at least in the final answer but preferably also in the intermediate steps. Provide your answers on a

Exam graded by: July 18, 2017 (to be filled in by course examiner)

(the marking period is 15 working days at most)

!

Every suspicion of fraud is reported to the Board of Examiners.

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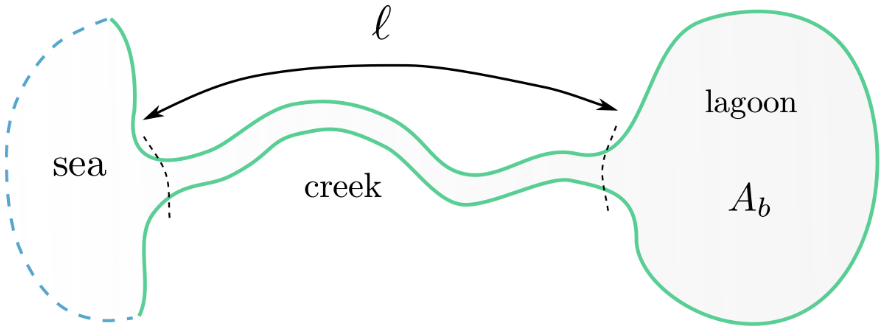
Question I (Tidal Lagoon)

A lagoon is connected to a coastal sea by means of a natural creek. At sea an M_2-tide prevails, with radian frequency $\omega = 1.4 \times 10^{-4}$ rad/s, causing a periodic water motion in the lagoon-creek system. See the figure below.



The response of the lagoon-creek system to the tidal forcing can be modelled using the so-called discrete system approach.

I.1$^{1.0}$ Describe the essential features of such a discrete system in terms of its constituting components, and explain why this approximation is valid only if the horizontal dimensions of the respective components are small relative to the local wave lengths.

For preliminary computations, the equations describing the discrete creek-lagoon system are linearized. The lagoon has a wet surface area A_b of $10\,\mathrm{km}^2$. The creek has a length ℓ of 12 km, a uniform cross section A_c of $325\,\mathrm{m}^2$, a hydraulic radius R of 2.5 m, and a resistance coefficient c_f of 0.004. The surface elevation amplitude of the tide *in the lagoon* (ζ_b) equals 0.20 m.

I.2$^{1.0}$ Quantify, by means of a computation, the relative importance of the resistance and the inertia terms, respectively, in the linear lagoon-creek system for the given tidal forcing.

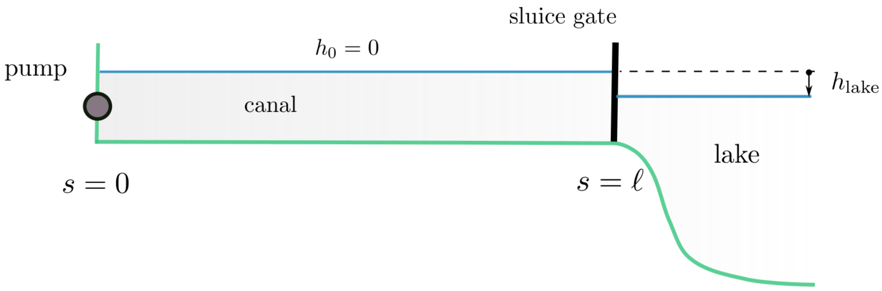
I.3$^{1.5}$ Compute the response factor of the lagoon-creek system and the corresponding surface elevation amplitude $\hat{\zeta}_s$ of the M_2-tide at sea.

The lagoon suffers from water quality problems due to waste disposal, clogging the creek, which hampers the tidal exchange flow. After a major clean-up of the creek, its conveyance cross section A_c equals $650\,\mathrm{m}^2$, the hydraulic radius R equals 5.0 m, and the resistance coefficient c_f is reduced to 0.002. The wet surface area of the lagoon, the length of the creek, and the tidal surface elevation amplitude at sea remain the same.

I.4$^{1.5}$ Compute the surface elevation amplitude in the lagoon after the clean-up and the corresponding relative increase of the discharge in the creek.

Question 2 (Drainage Canal)

A drainage canal, having a uniform rectangular cross section, is connected to a large lake by means of a sluice gate at $s = \ell$. At the other end of the canal ($s = 0$) a pumping station is situated. Initially, the sluice gate is closed and the pumps are off. The initial water level h_0 in the canal is chosen as reference level, which is above the water level h_{lake} in the lake, that is, $h_{\mathrm{lake}} < h_0 = 0$. See the figure below.



The canal has a cross section A_c of $25\,\mathrm{m}^2$ and a storage width B of $10\,\mathrm{m}$. At time $t = 0$ the sluice gate is opened partially while the pumps are still off. As a result, a negative translatory wave is issued into the canal with a height of $-0.25\,\mathrm{m}$.

II.1$^{1.0}$ Use linear wave theory to compute the discharge Q through the sluice gate, after it has been partially opened, and the flow velocity in the canal corresponding to the translatory wave; verify whether or not application of linear wave theory is justified in this case.

The boundary condition at the sluice gate ($s = \ell$) can be formulated as $h(\ell) = h_{\mathrm{lake}} + \xi|U(\ell)|U(\ell)/2g$, where $\xi = 20$ is the discharge coefficient of the sluice gate.

II.2$^{1.0}$ For the above situation with the sluice gate partially opened, compute the corresponding value of h_{lake}. (Use $h_{\mathrm{lake}} = -0.25\,\mathrm{m}$ in the remainder of this Question if you are not able to obtain an answer.)

The partial opening of the sluice gate and the corresponding wave problem will be investigated in more detail using the method of characteristics. To this end, the characteristic equations are formulated as follows: $\mathrm{d}h/\,\mathrm{d}U = \mp c/g = \mp\, 0.5\,\mathrm{m}/(\mathrm{m/s})$ along $\mathrm{d}s/\,\mathrm{d}t = \pm c = \pm\, 5.0\,\mathrm{m/s}$.

II.3$^{1.5}$ For $0 < t < 3t_\ell$, where t_ℓ is the travel time of translatory waves in the canal, construct the s,t-plane and the corresponding U,h-state diagram. Use the accompanying answer form. Plot the initial - and boundary conditions clearly and use Roman numbers to mark the different states.

At time $t = 3t_\ell$ the pumps are switched on leading to the boundary condition $U(0) = 0.50\,\mathrm{m/s}$ for $t \geq 3t_\ell$. The boundary condition at the sluice gate remains unchanged.

II.4$^{1.5}$ Complete the s,t-diagram and the U,h-state diagram for the time interval $3t_\ell < t < 6t_\ell$ taking into account the new boundary condition at $s = 0$ for $t > 3t_\ell$.

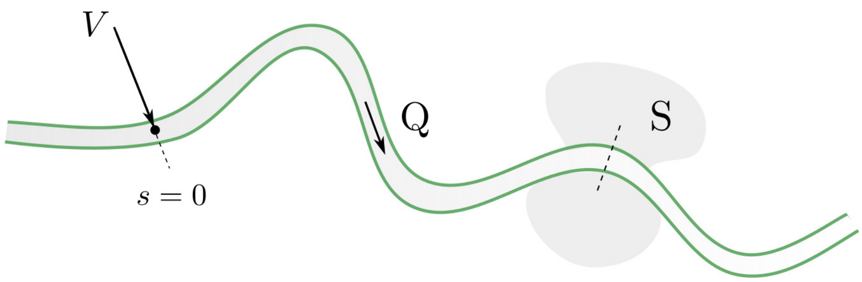
II.5$^{1.0}$ Give the Roman numbers corresponding to the regions in the s,t-diagram in which the activation of the pumps is noticeable, and use this as example to explain the concepts *domain of influence* and *domain of dependence*.

Question III (River Flood Wave)

A river reach is subject to frequent flooding as a result of heavy rainfall events in the upper catchment area of the river. To analyse the occurence of these floodings and to investigate possible mitigation measures a quasi-steady model for river flood waves is applied.

III.1$^{1.0}$ Explain which assumption is key to the quasi-steady approximation for river flood waves, and mention which important consequence this simplification has for the mathematical nature of the resulting wave equation.

The river reach of concern is schematized as a uniform channel having a cross section A_c of 1,200 m^2, a width $B = 2.5B_c = 325\,\mathrm{m}$, a constant bed slope $i_b = 2\times10^{-4}$ and a resistance coefficient $c_f = 0.003$. The hydraulic radius R is equal to A_c/B_c. A rainfall event is modelled as an instantaneous release of a volume water V in the river at a location $s = 0$. Downstream of this release point a settlement S is situated which is particularly vulnarable to flooding. See the figure below.



The top of the flood waves passes the settlement S at time $t_S = 8\,\mathrm{hrs}$ after the occurrence of rainfall.

III.2$^{1.0}$ Compute the distance of settlement S from the realease point $s = 0$.

For a particular rainfall event the released volume V amounts to $40 \times 10^6\,\mathrm{m}^3$.

III.3$^{1.5}$ Compute the maximum height of the resulting flood wave (relative to the undisturbed situation) when the top of the wave passes settlement S.

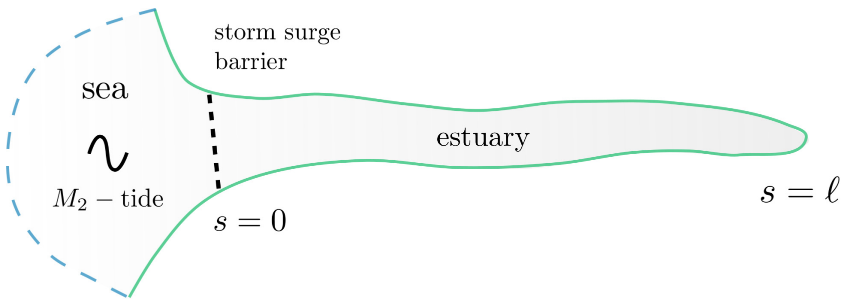
III.4$^{1.0}$ Explain why the maximum height computed in the previous question is less than the maximum height that occurs at S during the passage of the entire flood wave.

The flooding problems at S will stop if the maximum height of every flood wave that passes S is reduced with at least 35% compared to the height of the flood wave in the present situation. To achieve this reduction, it is proposed to increase the storage width B of the river while all other parameters are kept constant. For preliminary computations, the difference referred to in the previous question is negligible, and we may therefore assume that the maximum water height at S occurs simultaneously with the passage of the wave top.

III.5$^{1.5}$ Compute the minimum storage width B of the river that is required to save S from future floodings. Hint: both the high-water wave speed c_{HW} and the diffusion constant K depend on the storage width B.

Question IV (Storm Surge Barrier)

A semi-closed estuary is connected to a sea in which an M_2-tide prevails. During spring tides, the water level at the closed end of the estuary may become too high. Therefore, a storm surge barrier will be constructed at the entrance. In normal conditions the barrier will be open while during high spring tides it can be closed. See the figure below.



This question studies the tide in the estuary, before and after construction of the barrier, using the so-called harmonic method.

IV.1[1.0] Mention the necessary assumptions to reduce the one-dimensional shallow water equations to a set of equations describing the propagation of linear harmonic waves, and explain the conditions for which these approximations are valid. Give also an example of a tidal phenomenon that cannot be described with the harmonic method.

The estuary has a length ℓ of 45 km. The cross section is assumed uniform with a conveyance area A_c of 40,000 m^2, a storage width B of 2,500 m, and a resistance factor κ of $7 \times 10^{-5}\,\mathrm{s}^{-1}$. We will first consider the situation without storm surge barrier for which, during high spring tide, the amplitude of the surface level amplitude at the closed end $\hat{\zeta}_\ell$ amounts to 2.5 m.

IV.2[1.5] Compute the corresponding amplitude of the surface level elevation $\hat{\zeta}_0$ at the entrance of the estuary.

IV.3[1.0] Compute the corresponding amplitude of the tidal discharge $\hat{Q}_0$ at the entrance of the estuary.

Without storm surge barrier, the surface level elevation ζ_0 at the entrance of the estuary equals the surface level elevation ζ_s at sea. This changes however once the barrier has been constructed since associated expansion losses cause resistance, even if the barrier is open. This leads to a discharge relation of the form $\zeta_0 = \zeta_s - \xi |Q_0| Q_0 / (2gA_c^2)$, where $\xi = 2$ is the head loss coefficient of the open barrier.

IV.4[1.5] Explain how the discharge relation of the barrier can be linearized to obtain a boundary condition of the form $\zeta_0 = \zeta_s - \alpha Q_0$, and give the corresponding value for α using the answer of Question IV.3 as a first estimate for the discharge amplitude $\hat{Q}_0$.

IV.5[bonus point] Formulate the relation between the complex amplitudes of the discharge and the surface level elevation at the open entrance of a semi-closed basin, and explain how this can be combined with the above linearized boundary condition to obtain the reponse factor of the basin after construction of the storm surge barrier. A computation is not necessary, it suffices to explain the solution procedure.

Answers

Question I (Tidal Lagoon)

I.1 In a discrete system a conveying component and a storing component can be distinguished each of which have horizontal dimensions that are much smaller than the local wave length. As a consequence, the state in each of the components does not vary in space but varies in time only. In this way, the system as a whole is described by just two time dependent functions: the discharge $Q(t)$ in the conveying component (rigid-column approximation) and the water level $h_b(t)$ in the storing component (small-basin approximation).

I.2 The relative importance of inertia is quantified by the term ω^2/ω_0^2, in which the eigenfrequency $\omega_0 = \sqrt{gA_c/\ell A_b} = 1.6 \times 10^{-4}$ rad/s. For the M$_2$-tide this gives $\omega^2/\omega_0^2 = 0.74$. Since this value is not (much) smaller than 1 it can be concluded that the influence of inertia is significant. The relative importance of resistance is quantified by the term $\omega\tau$, in which the relaxation time $\tau = (8/3\pi)\chi\hat{Q}A_b/gA_c^2$. The discharge amplitude $\hat{Q} = \omega A_b\hat{\zeta}_b = 280\,\mathrm{m}^3/\mathrm{s}$, and $\chi = 1/2 + c_f\ell/R = 19.7$, which leads to $\tau = 4.52 \times 10^4\,\mathrm{s}$, and $\omega\tau = 6.33$. Since this value is (much) larger than 1 it follows that resistance is significant and also dominates the inertia term.

I.3 The complex response factor $\tilde{r} = \left(1 - \omega^2/\omega_0^2 + i\omega\tau\right)^{-1}$ (formula sheet) leading to an amplitude ratio $r = |\tilde{r}| = 1/\sqrt{(1-\omega^2/\omega_0^2)^2 + (\omega\tau)^2}$. Using the previously computed values for $\omega^2/\omega_0^2 = 0.74$ and $\omega\tau = 6.33$, respectively, gives $r = 0.16$. Using the definition $r \equiv \hat{\zeta}_b/\hat{\zeta}_s$ it follows that $\hat{\zeta}_s = 1.27\,\mathrm{m}$.

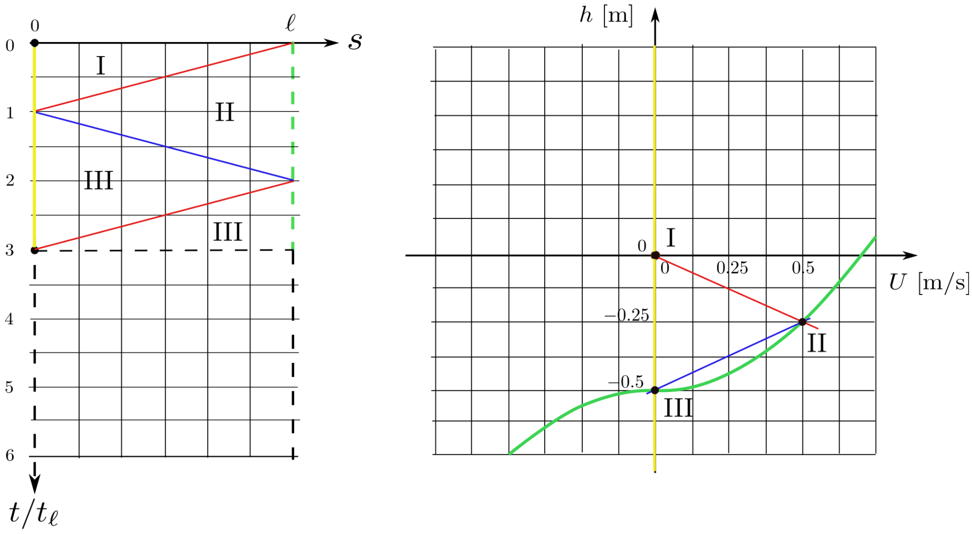
I.4 The amplitude ratio is given by $r = (1/\sqrt{2}\Gamma)\sqrt{\sqrt{\alpha^4 + 4\Gamma^2} - \alpha^2}$ in which $\alpha = 1 - \omega^2/\omega_0^2$ and $\Gamma = (8/3\pi)\chi\,(A_b/A_c)^2\,\omega^2\hat{\zeta}_s/g$. In the new situation $\omega_0 = 2.3\times10^{-4}$ rad/s, giving $\omega^2/\omega_0^2 = 0.37$ (influence moderate, not negligible), and $\alpha = 0.63$. To compute Γ we first calculate $\chi = 1/2 + c_f\ell/R = 5.3$ giving $\Gamma = 2.64$. This finally results in $r = 0.59$ and $\hat{\zeta}_b = r\hat{\zeta}_s = 0.74\,\mathrm{m}$. Since the discharge is proportional to the surface elevation amplitude in the basin the discharge has increased with $(0.73/0.20) \times 100\% = 325\ \%$ compared to the old situation.

Question II (Drainage Canal)

II.1 Use the wave height to discharge relation for a translatory wave travelling in the negative s-direction: $\delta Q^- = -Bc\,\delta h^-$. Using $c = \sqrt{gA_c/B} = 4.95\,\mathrm{m/s}$ and the given wave height $\delta\ h^- = -\ 0.25\,\mathrm{m}$ gives the corresponding dicharge $\delta Q^- = 12.4\,\mathrm{m}^3/\mathrm{s}$ which is also the discharge through the gate after opening (initial discharge equals zero).

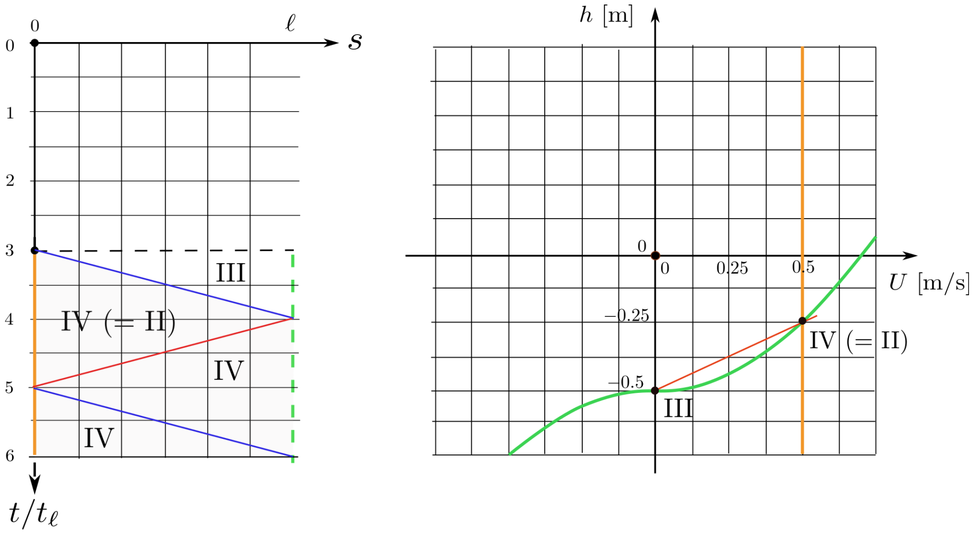
II.2 The flow velocity in the canal at $s = \ell$ immediately after opening the gate equals $U(\ell) = \delta Q^-/A_c = 0.50\,\mathrm{m/s}$. Using the discharge relation for the gate it follows that the head loss over the gate equals $\Delta h_{\mathrm{gate}} = h(\ell) - h_{\mathrm{lake}} = \xi|U|U/(2g) = 0.25\,\mathrm{m}$, where $h(\ell)$ is the water level in the canal at the gate immediately after opening the gate. Since $h(\ell) = h_0 + \delta h^- = -0.25\,\mathrm{m}$ this finally leads to $h_{\mathrm{lake}} = h(\ell) - \Delta h_{\mathrm{gate}} = -0.50\,\mathrm{m}$.

II.3 Initial condition: $U = 0,\ h = h_0 = 0$. Boundary conditions: $U(0) = 0,\ h(\ell) = -0.5\,\mathrm{m} + 10\,|U(\ell)|U(\ell)/g$. See the corresponding s,t-plane and the U,h-state diagram in the figure below (using $10/g \approx 1\ \mathrm{s^2/m}$).



s,t-plane U,h-state diagram

II.4 'Initial condition': $U = U_{\text{III}} = 0$, $h = h_{\text{III}} =$ -0.5 m. Boundary conditions: $U(0) = 0.5\,\text{m/s}$, $h(\ell) = -0.5\,\text{m} + 10\,|U(\ell)|U(\ell)/g$. See the continuation of the s,t-plane and the U,h-state diagram in the figure below.



s,t-plane U,h-state diagram

II.5 The influence of the pump is noticeable in the region of the s,t-plane that can be reached by a (positive) characteristic issued from the boundary where the pump is situated ($s = 0$) and after it has been switched on ($t > 3t_\ell$). This corresponds to the region labelled IV in the s,t-plane (shaded area in figure). Domain of influence (of a point): region of the $s,t-$plane that can be reached by characteristics issued from a point; domain of dependence (of a point): region of the s,t-plane from which the characteristics reaching a point are issued. Region IV is therefore within the region of influence of the boundary $s = 0$ for $t > 3t_\ell$.

Question III (River Flood Wave)

III.1 The quasi-steady approximation for river flood waves ignores the inertia term in the momentum equation (time derivative of the dicharge). The momentum equation then reduces to a discharge relation in which the discharge is expressed directly in terms of the cross-sectional parameters, the bed slope i_b and the free surface slope $\partial h/\partial s$. Substitution of this discharge relation into the continuity equation gives the high-water wave equation, the solutions of which characterize uni-directional wave propagation in the direction of the river flow with a speed comparable to the flow velocity; bi-directional propagation of gravity waves, having a much larger propagation speed, is singled out.

III.2 The distance from point S to the release point ($s = 0$) is computed from $s_S = t_s \times c_{HW}$. The high-water wave speed is $c_{HW} = \frac{3}{2}(B_c/B)U_u = 1.47\,\mathrm{m/s}$, using $U_u = \sqrt{gRi_b/c_f} = 2.46\,\mathrm{m/s}$ for the uniform flow velocity. It follows that $s_S = 42.34\,\mathrm{km}$.

III.3 When the top of the flood wave passes S its height is given by $\Delta d_{\mathrm{top,S}} = (V/B)/(\sqrt{2\pi}\,\sigma)$ (exponential factor of the general solution equals one at the wave top; $s - c_{HW}t = 0$). The standard deviation $\sigma = \sqrt{2Kt_S}$, and the diffusion constant $K = Q_u/(2i_bB)$. Using $Q_u = U_uA_c = 2{,}948\,\mathrm{m^3/s}$, results in $K = 22{,}680\,\mathrm{m^2/s}$ and $\sigma = 36{,}143\,\mathrm{m}$ (using $t_S = 8\,\mathrm{hrs}$), which finally gives $\Delta d_{\mathrm{top,S}} = 1.36\,\mathrm{m}$.

III.4 When the wave top passes, the exponential part of the general solution formula equals one, which is also its maximum; the time derivative of this factor is zero at this time instant. The factor $(V/B)/(\sqrt{2\pi}\sigma)$ is always decreasing in time, since it is proportional to $1/\sqrt{t}$. It follows that when the top passes S the depth at S decreases (falling stage of the local hydrograph); which can only happen after the depth at S has become maximum.

III.5 From the general solution formula it follows that, for a constant released volume V, the top height at S ($\Delta d_{\mathrm{top,S}}$) is inversely proportional to $B\sqrt{K\,t_{\mathrm{S}}}$, stated otherwise, $\Delta d_{\mathrm{top,S}} \propto 1/B\sqrt{K\,t_S}$. Since $K = U_uA_c/2i_bB$ and $t_S = s_S/c_{HW} = s_S\,B/(1.5B_cU_u)$ this proportionality can be written as $\Delta d_{\mathrm{top,S}} \propto 1/B\sqrt{A_c/B_c}$, for i_b and s_S constant. In this question the width B varies only while A_c and B_c remain unchanged giving $\Delta d_{\mathrm{top,S}} \propto 1/B$. For a reduction of the top height at S with 35 % we then have $\Delta d_{\mathrm{new}}/\Delta d_{\mathrm{old}} = B_{\mathrm{old}}/B_{\mathrm{new}} = 0.65$, from which it follows that the new storage width $B_{\mathrm{new}} = B_{\mathrm{old}}/0.65 = (325\text{ m})/0.65 = 500\,\mathrm{m}$.

Question IV (Storm Surge Barrier)

IV.1 The reduction of the one-dimensional shallow water equations to a linear equations comprises the folowing steps:

- the advective acceleration term is ignored, which is allowed if the wave height to depth ratio is small ($\ll 1$), and the undisturbed flow velocity is also small (the resulting Froude-number $Fr \ll 1$);
- the water level variations are sufficiently small to ignore the accompanying variations of the cross-sectional parameters (for instance A_c, B), this will usually be the case if the wave height is small relative to the water depth so that this condition matches with the previous one;
- the quadratic resistance term must be linearized, requiring the assumption of constant c_f and R and the replacement of the term $|Q|Q$ with $(8/3\pi)\hat{Q}Q$, assuming equal energy dissipation over a tidal cycle (Lorentz principle).

Nonlinear tidal phenomena cannot be decribed by the linear model. Examples are the tidal bore, the nonlinear deformation of a tidal wave, or mean net effects due to nonlinearity such as the set-up of the mean water level or the occurrence of a mean residual current.

IV.2 The surface elevation amplitude at the entrance is given by $\hat{\zeta}_0 = \hat{\zeta}_\ell\sqrt{\sinh^2\mu\ell + \cos^2 k\ell}$. To compute the wave number k and damping parameter μ first compute the wave number in absence of resistance: $k_0 = \omega/c_0 = 1.12 \times 10^{-5}\,\mathrm{rad/m}$, using the wave speed (no resistance) $c_0 = \sqrt{gA_c/B} = 12.53\,\mathrm{m/s}$. To include the effect of resistance compute first $\sigma = \omega/\kappa = 0.5$, and next $\delta = \frac{1}{2}\arctan\sigma = 0.23\,\mathrm{rad}$. The wave number is now given by $k = k_0/\sqrt{1 - \tan^2\delta} = 1.15 \times 10^{-5}\,\mathrm{rad/m}$ and the damping parameter $\mu = k\tan\delta = 2.71 \times 10^{-6}\ \mathrm{1/m}$. Using $\hat{\zeta}_\ell = 2.5\,\mathrm{m}$ and $\ell = 45\,\mathrm{km}$, we finally obtain $\hat{\zeta}_0 = 2.19\,\mathrm{m}$.

IV.3 The tidal discharge amplitude at the entrance is given by $\hat{Q}_0 = \hat{\zeta}_\ell Bc\cos\delta\sqrt{\sinh^2 k\ell + \sin^2 k\ell}$.

Using $c = \omega/k = 12.17\,\mathrm{m/s}$ and $\hat{\zeta}_\ell = 2.5\,\mathrm{m}$, and the previous values for k, μ and δ gives $\hat{Q}_0 = 37{,}740\,\mathrm{m^3/s}$.

IV.4 The discharge relation is effectively a local resistance term having the same mathematical format as the bed resistance through its proportionality to $|Q|Q$. Applying the principle of equal energy dissipation during a tidal cycle gives, for a sinusoidal variation of the discharge, $|Q_0|Q_0 \approx (8/3\pi)\hat{Q}_0 Q_0$. The coefficient α is therefore given by $\alpha = \xi(8/3\pi)\hat{Q}_0/(2gA_c^2)$. Using the given value for A_c, $\xi = 2$, and the previous discharge amplitude as a first estimate for Q_0, we obtain $\alpha = 2.04 \times 10^{-6}\,\mathrm{m/(m^3/s)}$.

IV.5 The given boundary condition provides a relation between the complex amplitudes of the discharge and the surface level elevation at the entrance ($s = 0$). This is one equation with two unknowns. To obtain a solution we need another (independent) relation between $\tilde{Q}_0$ and $\tilde{\zeta}_0$. For a uniform basin, closed in one side, the relation between the complex amplitudes of the discharge and the surface level elevation in the entrance reads $\tilde{Q}_0 = \tilde{\zeta}_0\, Bc\, \cos\delta \mathrm{e}^{\mathrm{i}\delta} \tanh \mathrm{p}\ell$ (where $p = \mu + ik$). Substitution into the boundary condition, and rewriting, gives $\tilde{\zeta}_0 = \tilde{\zeta}_s/(1+\alpha Bc\, \cos\delta\, \mathrm{e}^{\mathrm{i}\delta} \tanh \mathrm{p}\ell)$. For a given surface elevation amplitude at sea ($\tilde{\zeta}_s$) the corresponding amplitude at the entrance ($\tilde{\zeta}_0$ landward of the barrier) can be computed after which the solution proceeds in the same manner as for an 'open' estuary. The general effect of a barrier will be a reduction of the water level amplitude in the basin which was also experienced after the construction of the Eastern Scheldt barrier. The ecological problems this has caused even continue today.

Faculty Civil Engineering and GeoSciences

TU Delft

Exam	CTB3350 (BSc) CIE3310-09 (MSc)
	Open Channel Flow
Total number of pages	6 pages
Date and time	June 28, 2017 at 9:00 - 12:00 hours
Responsible lecturer	R. J. Labeur

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of questions: 4 (of which 4 open questions and 0 multiple choice questions)

Max. number of points to be granted: 22

☐ all questions have equal weight
☑ questions differ in weight (the weight is mentioned per question, or is given in an overview)

Use of tools and sources of information (to be filled in by course examiner)

Not allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with pencil.
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ books ☐ notes ☐ dictionaries ☐ syllabus
☑ formula sheets (see also below under 'additional information') ☑ calculators ☐ computers
☐ ...

Additional information (if necessary to be filled in by the examiner)

The number of points that can be obtained for each subquestion is shown next to the respective question number. When grading your exam, special attention will be given to a correct application of the underlying theory and a systematic approach to the problem. Provide your answers with a short description of the chosen solution strategy. If you need to make assumptions or simplifications briefly explain why it is allowed to do so. Where a previous answer is needed in a subsequent question, but which you were not able to compute, assume a plausible numerical value. Numerical answers should contain the correct dimensional units, at least in the final answer but preferably also in the intermediate steps. Provide your answers on a

Exam graded by: July 18, 2017 (to be filled in by course examiner)

(the marking period is 15 working days at most)

!

Every suspicion of fraud is reported to the Board of Examiners.

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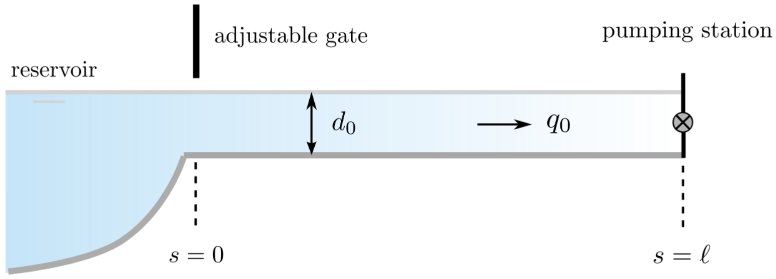
Question I (Primary irrigation canal)

Water from a storage reservoir is supplied to an irrigation network by means of a primary canal having a horizontal bottom and a rectangular cross-section with uniform width. The canal is connected to the reservoir by means of an adjustable gate. The other end of the canal is connected to a pumping station. Initially, the gate is fully open and the pump is on with a steady, uniform flow state in the canal with a depth $d_0 = 2.50\,\mathrm{m}$ and a specific discharge $q_0 = 0.50\,\mathrm{m^2/s}$. See the figure below.



Manipulation of the gate and/or the pumping station in the above situation will cause translatory waves in the canal, superimposed on this initial flow state.

I.1[1] Explain why for *low* waves it is justified in this case to neglect the advective acceleration term in the momentum equation, even though the initial flow velocity is different from zero.

At time $t = 0$ the gate is closed *partially* while the water level in the reservoir remains constant. This results in a negative translatory wave in the canal with a height δh^+ of -0.05 m.

I.2[1] Use linear wave theory to compute the specific discharge q through the gate directly after it has been closed partially.

The boundary condition at the partially closed gate reads

$$d(0) = d_0 - \xi \frac{|U(0)|\,U(0)}{2g} \tag{1}$$

where $d(0)$ and $U(0)$ are the water depth and flow velocity in the canal at $s = 0$, respectively, and ξ is the discharge coefficient of the gate.

I.3[1] Compute the discharge coefficient ξ for which the issued wave has a height δh^+ of -0.05 m (i.e. the situation of Question I.2).

We will now study the partial closure of the gate using the method of characteristics. To this end the characteristic equations are linearized as follows:

$$\frac{\delta d}{\delta U} = \mp \frac{c_0}{g} = \mp 0.5\,\frac{\mathrm{m}}{\mathrm{m/s}} \quad \text{for} \quad \frac{\mathrm{d}s}{\mathrm{d}t} = \pm c_0 = \pm 5\,\mathrm{m/s} \tag{2}$$

The boundary condition at $s = 0$ (gate) is discharge relation (1) with the discharge coefficient ξ as computed in Question II.3. (Use $\xi = 50$ if you were not able to obtain an answer to this question.) The boundary condition at $s = \ell$ (pumping station) reads $U(\ell) = q_0/d_0 = 0.2\,\mathrm{m/s}$.

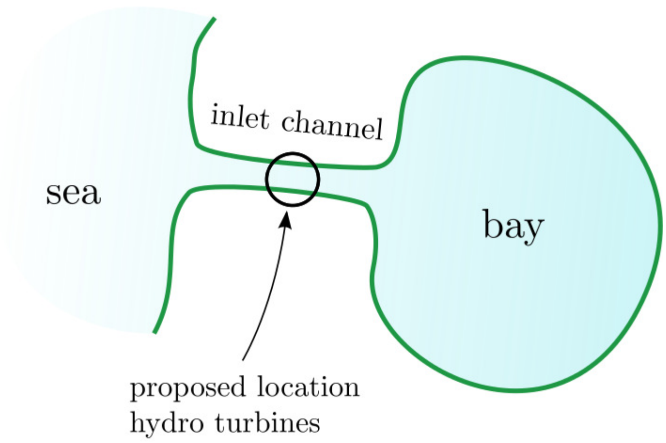
I.4[1.5] For the time interval $0 < t < 4t_\ell$, where t_ℓ is the travel time of waves in the canal, construct the s, t plane and the U, d state diagram using the above initial and boundary conditions. Provide these graphs with the relevant numbers specifying the initial and boundary conditions.

Some time after the partial closure of the gate the flow in the canal becomes steady. Next, after the required amount of water has been delivered, the gate is closed fully and the pump is switched off simultaneously.

I.5[1.5] Compute the heights of the translatory waves issued from the respective channel ends.

Question II (Tidal energy basin)

A natural bay is connected to a tidal sea by a narrow inlet channel. At sea an M_2 tide prevails. It is to be investigated whether or not this tidal bay-inlet system can be used as a source of sustainable energy by constructing hydro-turbines in the inlet channel. See the figure below for an impression of the situation.



The bay-inlet system can be modeled as a so-called *discrete system*.

II.1[1] Explain thé essence of this model approximation, and describe the conditions for which it is valid.

We will study first the bay-inlet system in its original configuration. The bay has a surface area A_b of 18 km^2. The inlet channel has a length ℓ of 800 m and a conveying cross-section A_c of 1400 m^2.

II.2[1] Determine quantitatively whether or not inertia is important in this bay-inlet system.

The inlet channel has a hydraulic radius R of 8 m, and a bed resistance coefficient c_f of 5×10^{-3}. The tidal amplitude at sea ($\hat{\zeta}_s$) amounts to 2.50 m.

II.3[1.5] Compute the amplitude of the surface level $\hat{\zeta}_b$ in the bay and the amplitude of the tidal discharge $\hat{Q}$ in the inlet channel.

The installation of a particular number of turbines in the inlet channel will *increase* the resistance parameter χ of this channel with an amount $\chi_t = 0.1 \times N_t$, where N_t is the number of turbines installed. As a consequence, the tidal water level amplitude $\hat{\zeta}_b$ in the bay will decrease once the turbines have been installed. However, for environmental reasons, $\hat{\zeta}_b$ should remain larger than 1.20 m.

II.4[1.5] Compute the maximum number of turbines that can be installed in the inlet channel that satisfies this environmental criterion.

The nominal energy production P_t of the turbines is proportional to $\rho\, \chi_t\, \hat{Q}^3/A_c^2$. A computation reveals that the energy production has a maximum if the number of installed turbines equals 30.

II.5[1] Explain why the number of turbines has an optimum regarding energy production; stated otherwise, why will P_t decrease if N_t is either smaller or larger than the optimum number of 30?

Question III (High-water wave)

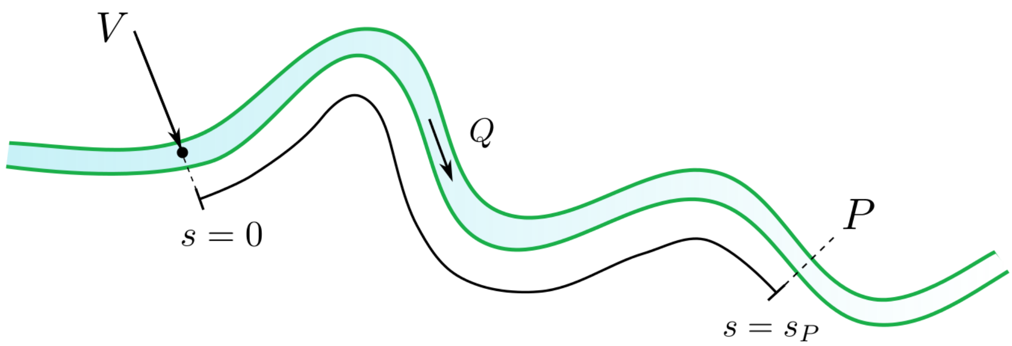
An elongated river reach is subject to frequent flooding due to heavy rainfall in the upper catchment area of the river. The character of the resulting high-water waves is initially diffusive, becoming kinematic after they have traveled some distance down the river.

III.1^{1} Describe the similarities and differences between these two types of high-water wave, and explain why a high-water wave will ultimately behave as a kinematic wave, provided the river reach in which it propagates is sufficiently long.

For preliminary investigations, the considered river reach is schematized as a uniform channel having a width $B = 3B_c = 400\,\mathrm{m}$, a bed slope $i_b = 2 \times 10^{-4}$, and a resistance coefficient $c_f = 4 \times 10^{-3}$. The hydraulic radius $R \approx d$. In the undisturbed (uniform) situation the discharge Q_u in the river equals $1350\,\mathrm{m}^3/\mathrm{s}$.

III.2^{1} Compute the initial flow velocity and the propagation speed of high-water waves in the river, based on these initial values.

A rainfall event is simulated as an instantaneous release of a volume water V in a point $s = 0$. We will study the effect of rainfall on the flow state in the river using the diffusion model for high-water waves and the previous schematization of the river reach. See the figure below.



For a particular rainfall event, the released water volume V amounts to $20 \times 10^6\,\mathrm{m}^3$.

III.3^{1} Compute the peak height $\Delta d_{\mathrm{peak},P}$ of the resulting high-water wave as the wave passes a point P having a streamwise coordinate s_P of $75\,\mathrm{km}$.

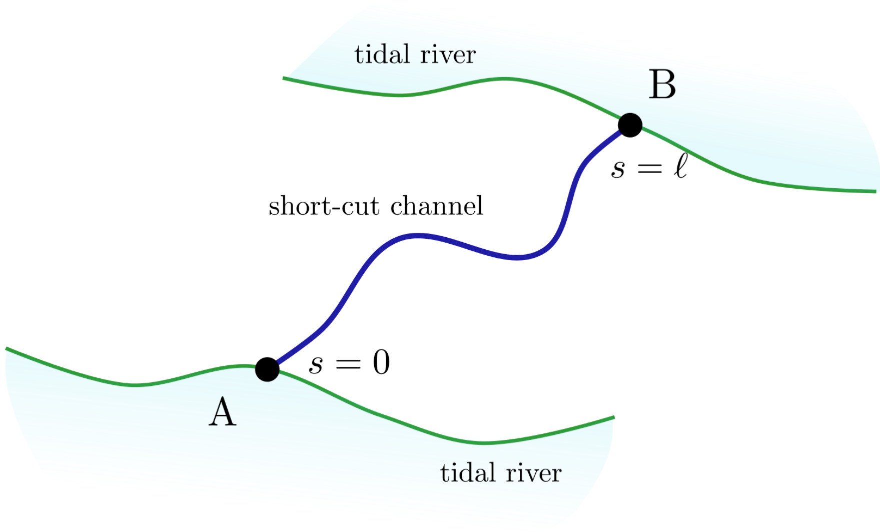
III.4$^{1.5}$ Derive an analytical expression for the peak height $\Delta d_{\mathrm{peak},P}$ in an arbitrary point P, as a function of the streamwise coordinate $s_P > 0$ of point P.

Problems occur where the depth increment during the high-water peak exceeds a critical value $\Delta d_{\max}$. For the given rainfall event, this is the case for $0 < s < s_{\max}$, where $s_{\max} = 60\,\mathrm{km}$. By increasing the storage width B of the river, $s_{\max}$ should be reduced to $15\,\mathrm{km}$. The other cross-sectional parameters of the river are to be left unchanged.

III.5$^{1.5}$ Compute the required storage width B to achieve this goal.

Question IV (Tidal short-cut channel)

Two large tidal rivers 'A' and 'B', respectively, are naturally connected by a narrow short-cut channel. In both tidal rivers an M_2 tide prevails causing a tidal flow in the short-cut. See the figure below.



The short-cut channel has a length ℓ of 55 km. Its cross-section is approximately uniform with a conveying cros-section A_c of 800 m^2 and a storage width $B =$ of 300 m.

II.1[1] Quantify whether or not the tidal flow in the short-cut channel can be modeled using the *rigid column approximation*.

The tidal flow in the short-cut channel is modeled using the harmonic method. The corresponding boundary conditions for the complex water level amplitude at ends A ($s = 0$) and B ($s = \ell$)of the short-cut are given by $\tilde{\zeta}_0$ and $\tilde{\zeta}_\ell$, respectively.

IV.2[1] Explain how the general solution formula for the complex amplitude of the water level (given in the formula sheet) can be reworked to solve the above problem. You may either derive this solution, or describe the solution process in words.

The short-cut channel will be closed by means of a dam. The dam can be located at one of the ends A or B, or half way the channel ($s = \frac{1}{2}\ell$). The location having the smallest water level amplitudes at the location of the dam is preferred (since this gives the lowest construction costs).

Additional information: relative resistance short-cut channel: $\sigma = 0.5$; water level amplitudes at channel ends: $\hat{\zeta}_0 = 1.20\,\mathrm{m}$, $\hat{\zeta}_\ell = 0.80\,\mathrm{m}$.

IV.3[2] Determine the preferred dam location by comparing the water level amplitudes at the dam for the various alternatives. Assume that for each alternative the relative resistance equals the given value in the original configuration.

IV.4[1] Interpret the results found in the previous question by comparing the respective values of the dimensionless parameters that determine the amplification factor of a semi-closed basin.

So far, it has been assumed that the relative resistance σ of the short-cut channel is the same no matter where the dam is located. For more accurate computations however, σ should be based on the so-called *representative* discharge amplitude $\hat{Q}_{\mathrm{rep}}$ of the channel.

IV.5[1] Which principle is used to determine the representative discharge amplitude of a channel in the framework of the harmonic method?

Answers

Question I (Primary irrigation canal)

I.1[1] The (nonlinear) advective acceleration term can be neglected with respect to the local acceleration term since their ratio is approximately equal to the wave height to depth ratio, which is negligeable for low waves. Second, the influence of the flow velocity on the wave speed ($U + c$), which is also an effect of the advective acceleration term, can be ignored since $U_0 \ll c_0$ (calculation: $U_0 = q_0/d_0$ =0.20 m/s $\ll$ $c_0 = \sqrt{gd_0} = 4.95$ m/s).

I.2[1] Use the wave height to discharge relation for a linear wave in positive direction: $\delta q^+ = c_0 \delta h^+ =$ -0.25 m^2/s. This discharge variation is superimposed on the intial discharge q_0 resulting in a total discharge through the gate after partial closure of $q(0) = q_0 + \delta q^+ = +0.25$ m^2/s.

I.3[1] The depth $d(0)$ in the canal ($s = 0$) immediately after the partial closure equals $d(0) = d_0 + \delta h^+ =$ 2.45 m. The flow velocity $U(0)$ in the canal at $s = 0$ directly after closure equals approximately $U(0) = q(0)/d_0 =$ (0.25 m^2/s)/(2.50 m) = 0.10 m/s. Substituting these values in the discharge relation for the gate gives $\xi = 96$.

I.4[1.5] See the s, t plane and U, d state diagram below.

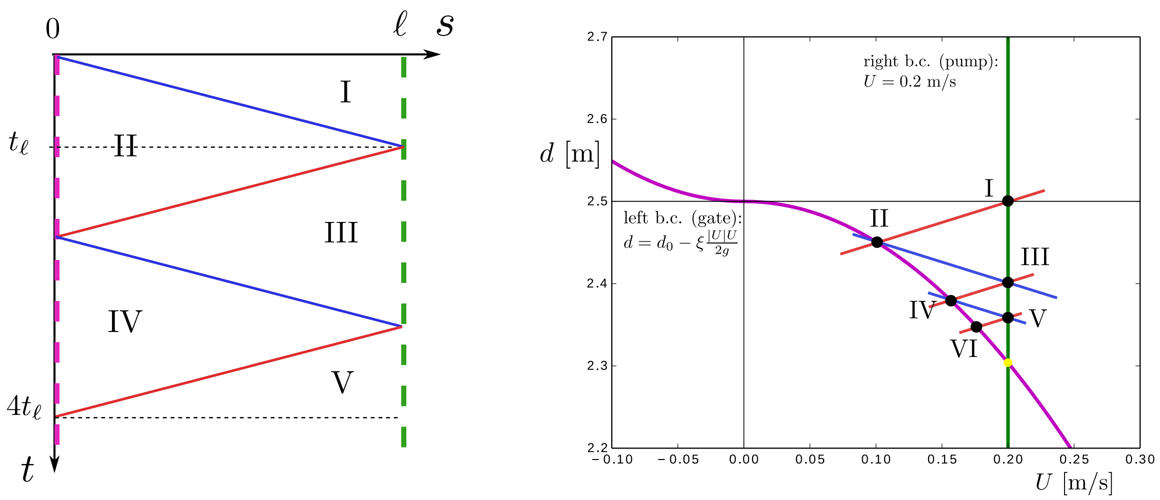


Figure 1: s, t plane (left) and U, d state diagram (right)

I.5[1.5] After the partial closure of the gate the flow state in the canal become steady with a discharge $q_\infty = q_0$=0.50 m^2/s. Closing the pump then gives (at $s = \ell$) a left going wave with $\delta q^- = -q_\infty =$ -0.50 m^2/s, resulting in a corresponding wave height $\delta h^- = -\delta q^-/c_0 = 0.10$ m. Full closure of the gate (at $s = 0$) then gives a right going wave with $\delta q^+ = -q_\infty =$ -0.50 m^2/s, resulting in a wave height $\delta h^+ = \delta q^+/c_0 =$ -0.10 m.

Question II (Tidal energy basin)

II.1[1] Thé essence of the discrete model approximation: the water system has a component wich only conveys water, and another component which only stores water. Furthermore, both components have horizontal dimensions that are small with respect to the local wavelength (a ratio of 5% is a usual upper limit). As a result, the state of system can be described by two variables that depend on time t only, being the water level $h_b(t)$ in the storing part and the discharge $Q(t)$ in the conveying part.

II.2[1] The relative magnitude of the inertia (acceleration) term is given by the ratio ω^2/ω_0^2, where $\omega_0 = \sqrt{gA_c/\ell A_b}$ is the eigenfrequency of the system. Using the given parameters it follows that $\omega_0 = 9.8 \times 10^{-4}$ rad/s, and $\omega^2/\omega_0^2 = 0.02 \ll 1$. The inertia term can therefore be neglected for this system.

II.3[1.5] For a discrete system without inertia the response factor for the tidal water level amplitude in the bay is given by $r = (1/\sqrt{2}\Gamma)\sqrt{\sqrt{4\Gamma^2+1}-1}$ where $\Gamma = (8/3\pi)\chi(\omega A_b/A_c)^2\hat{\zeta}_s/g$. The resistance factor of the entrance channel is given by $\chi = \frac{1}{2} + c_f\ell/R = 1.0$. Using this, the given parameters characterizing the channel-bay system, and the water level amplitude at sea we obtain $\Gamma = 0.70$. Substitution into the equation for the response factor gives $r = 0.87$ and $\hat{\zeta}_b = 2.17\,\text{m}$. The corresponding discharge amplitude in the channel equals $\hat{Q} = \omega A_b\hat{\zeta}_b = 5465\,\text{m}^3/\text{s}$.

II.4[1.5] We first compute χ for the given minimum water level amplitude in the bay. The corresponding response factor $r_{\min} = \hat{\zeta}_{b,\min}/\hat{\zeta}_s = 0.48$. Using $r_{\min} = 1/\sqrt{1+(\omega\tau)^2}$ (formula sheet) results in $\omega\tau = 1.83$ and $\tau = 13054\,\text{s}$. Since $\tau = (8/3\pi)\chi\hat{Q}A_b/gA_c^2 =$ and the disharge amplitude corresponding to $r_{\min}$ equals $\hat{Q} = \omega A_b r_{\min}\hat{\zeta}_s = 3024\,\text{m}^3/\text{s}$, we find $\chi = 5.43$. Since χ depends on the number of turbines, as follows: $\chi = \frac{1}{2} + c_f\ell/R + 0.1N_t$ the maximum number of turbines equals $N_t = 10 \times (\chi - \frac{1}{2} - c_f\ell/R) = 44$. (It is also possible to the compute the value of Γ corresponding to $r_{\min}$ using $r = 1/\sqrt{1+(\Gamma r)^2}$, after which χ is obtained from Γ.)

II.5[1.5] Initially, the bay-channel system is resistance dominated and this remains so (even more) after installation of the turbines. The more turbines are installed, the larger will be χ, the larger will be the tidal damping, and the smaller will be the discharge amplitude in the channel. The energy production is proportional to the product of the total turbine resistance (χ_t), which *increases* with the number of turbines, and a term that *decreases* with the number of turbines ($\hat{Q}^3$). The production therefore has a maximum for a specific number of turbines. (Fortunately, in the case, this optimal number is within the allowable range.)

Question III (High-water wave)

III.1[1] Both the kinematic wave model the diffusive wave model are *quasi-steady* approximations of the shallow water equations, that is, the local acceleration term is neglected in the momentum equation. In addition, the kinematic wave is based on the assumption that the flow is also *quasi-uniform*, that is, the discharge depends on the local depth and the bed slope only. In the diffusive wave approach the latter simplification is not made and the discharge is therefore dependent on the local surface level gradient, which includes the depth gradient as well. This leads to the so-called hysteresis; for the same depths, the discharge in the leading edge of the wave is larger than the discharge in the trailing edge due to the different depth gradients. This causes damping and flattening of the diffusive wave. In contrast, the kinematic wave will preserve its height (although it may deform since the wave speed c_{HW} depends on the local water depth). As the flattening of a diffusive wave proceeds, the influence of the depth gradient, causing the diffusive behaviour, becomes less and less important and will ultimately disappear leading to a kinematic wave.

III.2[1] First, compute the uniform flow depth d_u. Using the discharge relation for uniform flow $Q_u = A_c\sqrt{gRi_b/c_f}$, $R = d$ and $A_c = B_c d$ gives $Q_u = B_c d_u\sqrt{gd_u i_b/c_f}$. This leads to a water depth $d_u = (c_f Q^2/gi_b B_c^2)^{1/3} = 5.93\,\text{m}$. The corresponding flow velocity amounts to $U_u = Q_u/B_c d = 1.71\,\text{m/s}$. Finally, the high-water wave speed $c_{HW} = \frac{3}{2}(B_c/B)U_u = 0.85\,\text{m/s}$.

III.3[1] The peak height of the high-water wave equals $\Delta d_{\text{peak}} = (V/B)/\sqrt{2\pi}\,\sigma_s(t)$, where the standard deviation $\sigma_s(t) = \sqrt{2Kt}$, in which t is the elapsed time. The diffusion coefficient $K = Q_u/(2i_b B) = 8438\,\text{m}^2/\text{s}$. If the wave peak passes point P the elapsed time (denoted with t_P) equals $s_P/c_{HW} = 87918\,\text{s}$

(24 h 25 min), resulting in a corresponding value for $\sigma_s = 38.52\,\mathrm{km}$. The peak height is then equal to $0.52\,\mathrm{m}$.

III.4[1.5] Carrying out the same steps as in the previous answer, using symbols instead of numerical values gives: $\sigma_s(t_P) = \sqrt{2Kt_P} = \sqrt{2(Q_u/2Bi_b)\,s_P/c_{HW}}$. Using $c_{HW} = \frac{3}{2}(B_c/B)U_u$ and $Q_u = U_u B_c d$, this can be simplified to $\sigma_s(t_P) = \sqrt{\frac{2}{3}\,s_P\,d/i_b}$. For the peak height in point P then results $\Delta d_{\mathrm{peak},P} = (V/B)/\sqrt{\frac{4}{3}\pi\,s_P\,d/i_b}$.

III.5[1.5] The maximum depth $d_{\max}$ that used to occur at location $s_1 = 60\,\mathrm{km}$ should now occur at a location $s_2 = 15\,\mathrm{km}$. Using the formula derived in the previous question, it follows that $(V/B_1)/\sqrt{\frac{4}{3}\pi\,s_1\,d_u/i_b} = (V/B_2)/\sqrt{\frac{4}{3}\pi\,s_2\,d_u/i_b}$ where B_1 and B_2 are the storage widths in the old and new situation, respectively. Cancelling the common factors, $B_1/B_2 = \sqrt{s_2/s_1}$. Since $s_2 = \frac{1}{4}s_1$, the new storage width $B_2 = 2B_1 = 800\,\mathrm{m}$.

Question IV (Tidal short-cut channel)

IV.1[1] The rigid column approximation may be applied if the length ℓ of a channel is smaller than 1/20th of the local wave length. The local wave length equals $L_0 = Tc = 2\pi c_0/\omega$. Using $c_0 = \sqrt{gA_c/B} = 5.11\,\mathrm{m/s}$, we obtain L_0 =230 km which is (much) smaller than 20 times the channel length (1100 km). The rigid column approximation is therefore not applicable here.

IV.2[1] The general solution for a damped harmonic wave in a uniform channel reads (formula sheet): $\tilde{\zeta}(s) = C^+ \exp -ps + C^- \exp ps$. Substituting the boundary conditions in $s = 0$ and $s = \ell$, respectively gives $C^+ + C^- = \tilde{\zeta}_0$ and $C^+ \exp -p\ell + C^- \exp p\ell = \tilde{\zeta}_\ell$. Since $\tilde{\zeta}_0$ and $\tilde{\zeta}_\ell$ are known this results in two equations of which the two unkowns, C^+ and C^-, can be solved.

IV.3[2] Computation of two amplification factors (r), for a dam in one of the end point of the channel, and a dam in the middle of the channel, respectively, provides all the necessary information. For a dam in one of the end points this gives $r_1 = 1/\sqrt{\sinh^2 \mu\ell + \cos^2 k\ell}$ and for a dam in the middle the amplification factor equals $r_2 = 1/\sqrt{\sinh^2 \mu\frac{1}{2}\ell + \cos^2 k\frac{1}{2}\ell}$. To compute k and μ, determine first $k_0 = \omega/c_0 = 2.75 \times 10^{-5}\,\mathrm{rad/m}$. Next, $\delta = \frac{1}{2}\arctan\sigma = 0.23\,\mathrm{rad}$. It follows that $k = k_0/\sqrt{1 - \tan^2\delta} = 2.82 \times 10^{-5}\,\mathrm{rad/m}$, and $\mu = k\tan\delta = 6.65 \times 10^{-6}\,1/\mathrm{m}$. The resulting values for the response factors are $r_1 = 2.67$ and $r_2 = 1.36$, respectively. Using the response factors the water level amplitudes at the dam location can be easily computed, they are summarized in the table below. (Note that in each case there are two amplitudes, one on each side of the dam.)

	dam in A	dam in B	dam in middle
'A side' of dam	$\hat{\zeta}_0 = 1.20\,\mathrm{m}$	$r_1\hat{\zeta}_0 = 3.20\,\mathrm{m}$	$r_2\hat{\zeta}_0 = 1.63\,\mathrm{m}$
'B side' of dam	$r_1\hat{\zeta}_\ell = 2.14\,\mathrm{m}$	$\hat{\zeta}_\ell = 0.80\,\mathrm{m}$	$r_2\hat{\zeta}_\ell = 1.09\,\mathrm{m}$

Table 1: Water leval amplitudes at the dam

It follows that the optimal dam location is in the middle of the channel since this gives the smallest maximum water level amplitude at the dam location.

IV.4[1] Considering the given value for σ, the influence of the resistance in this system is relatively small, though not negligeable. It will not dominate the response. The other dimensionless parameter determining the response is the basin length to wave length ratio ℓ/L_0 (alternatively this may be expressed as $k_0\ell$). For a dam situated in an end point $\ell/L_0 \approx 0.23$, which is (very) near to the first resonant wave mode ($\ell/L_0 = 0.25$). Combined with the limited influence of the resistance, the corresponding repsonse factor is relatively large. For the dam in the middle, the situation is away from the resonant mode, reducing the amplification factor considerably. However, due to the small influence of the resistance it is still larger than 1.

IV.5[1] The linear resistance factor κ of the channel is based on a constant value for the discharge amplitude $\hat{Q}$ in the channel (or a constant velocity amplitude $\hat{U}$). In general, the discharge amplitude will vary along the channel. (Otherwise the rigid column approximation could have been used!). A representative constant value $\hat{Q}_{\mathrm{rep}}$ can be derived by equating the exact energy dissipation in the channel to the dissipation caused by the representative discharge amplitude, giving: $\int_0^\ell |\hat{Q}(s)|^3 \, \mathrm{d}s = \ell \, \hat{Q}_{\mathrm{rep}}^3$. This approach is conceptually similar to Lorentz' linearization of the quadratic resistance term which considers the total dissipation over a tidal period.

Delft University of Technology

Faculty of Civil Engineering and Geosciences

Section of Environmental Fluid Mechanics

Exam CTB3350 / CIE3310-09

'Open Channel Flow'

Tuesday June 23rd 2015, 9:00 to 12:00 hours

The number of points that can be obtained for each subquestion is shown next to the respective question number. When grading your exam, special attention will be given to a correct application of the underlying theory and a systematic approach to the problem. Provide your answers with a short description of the approach chosen. If you need to make assumptions or simplifications briefly explain why it is allowed to do so. Where a previous answer is needed in a subsequent question which you were not able to compute, assume a plausible numerical value. Numerical answers should contain the correct dimensional units, at least in the final answer but preferably also in the intermediate steps.

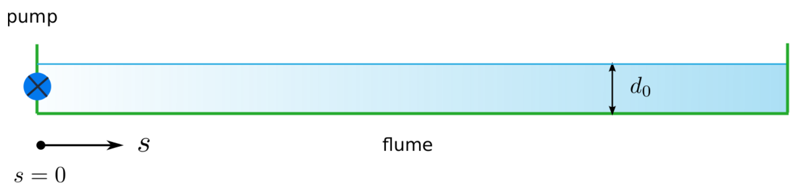
Unless explicitly stated otherwise use: mass density water $\rho_w = 1000\,\mathrm{kg/m^3}$, gravitation $g = 9.8\,\mathrm{m/s^2}$ and frequency M_2-tide $= 1.4 \times 10^{-4}\,\mathrm{rad/s}$.

Provide your answers on a separate form for each question.

Don't forget your name and student number on each form!

Question I (compression wave)

Consider a laboratory flume with a constant width $B = B_c$ and horizontal bottom. One end of the flume is closed while at the other end ($s = 0$) a pump is situated. Initially, the pump is off and a situation of rest prevails in the flume with initial depth $d_0 = 0.25\,\mathrm{m}$.



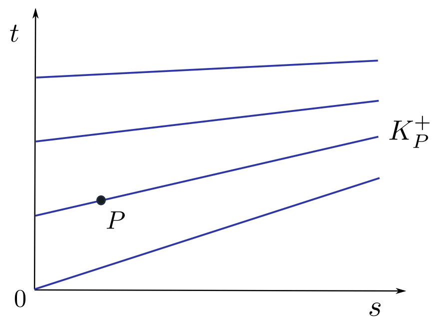
At time $t = 0$ the pump is switched on with the specific discharge q gradually increasing in time to $0.20\,\mathrm{m}^2/\mathrm{s}$, which results in a singular translatory wave in the flume.

I.1[1] Compute the total height of the translatory wave using linear wave theory and comment whether application of linear wave theory is justified in this case or not.

Generally, Riemann-invariants $R^+ = U + 2c$ are constant along positive characteristics, but for a singular wave traveling into an undisturbed region (as in this case) both the flow velocity U *and* c ($= \sqrt{gd}$) are constant along positive characteristics.

I.2[1] Explain why both U and c are constant along positive characteristics in such cases.

The figure below shows a set of positive characteristics corresponding to the translatory wave in the flume, obtained from the fully non-linear solution. The characteristics are perfectly straight lines, even though the problem and its solution are non-linear. Furthermore, these positive characteristics converge.



I.3[1] Explain why these positive characteristics are straight, converging lines.

Consider a point P on the characteristic K_P^+ with a depth $d_P =$ of $0.30\,\mathrm{m}$.

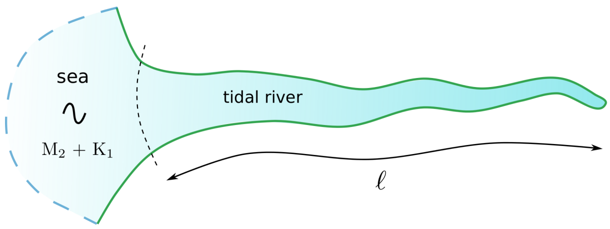
I.4[1.5] Compute the velocity U_P in point P, and the characteristic slope $(\mathrm{d}s/\mathrm{d}t)$ of K_P^+.

The flume is sufficiently long for the translatory wave to collapse and break, after which it continues as a bore propagating with a speed $c_{\text{bore}} = \sqrt{g(d_1/d_0)(d_1 + d_0)/2}$, in which d_1 is the depth on the trailing edge of the bore.

I.5[1.5] Compute the height $(d_1 - d_0)$ of the bore, and explain the difference(s) with the answer of Question I.1.

Question II (tidal river)

A tidal river is openly connected to a sea. The tide propagating into the river has two components, M_2 (semi-diurnal) and K_1 (diurnal), respectively. See the figure below.



II.1[1] Describe, in general terms, the influence of the frequency on the propagation of *progressive* harmonic waves in a uniform channel with constant linear friction factor.

For preliminary analyses of the *standing* wave in the tidal river the governing shallow water equations are linearized. Furthermore, the tidal river is schematized as a prismatic channel with a length ℓ of 135 km, a conveying cross-section A_c of $2.5 \times 10^4\,\mathrm{m}^2$, and a storage width B of 5 km.

II.2[1] Determine the smallest frequency (ω_0) for which, in absence of resistance, the periodic wave motion in the tidal river becomes resonant.

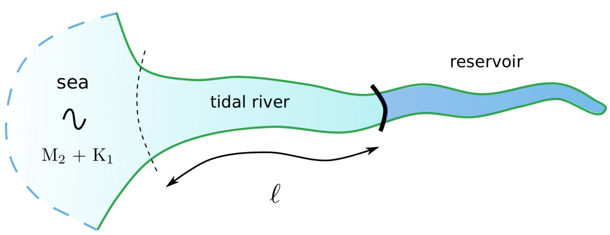
The tidal components have frequencies of, respectively, $\omega_{M2} = 1.4 \times 10^{-4}$ rad/s and $\omega_{K1} = 0.7 \times 10^{-4}$ rad/s. The friction factor κ of the tidal river amounts to $5 \times 10^{-5}\,\mathrm{s}^{-1}$.

II.3[1.5] For each of the tidal components, compute the dimensionless parameter(s) that determine the response of the river to tidal forcing, and use them to compare (qualitatively) the amplification factors of the respective components.

The surface elevation amplitude of the K_1 component at sea amounts to $\hat{\zeta}_{s,K1} = 0.35$ m.

II.4[1.5] Compute the surface amplitude of the K_1 tide at the closed end of the river.

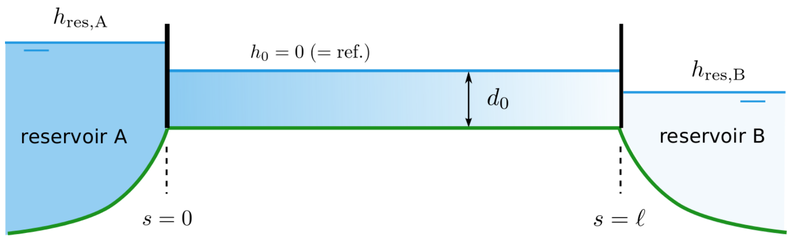
In the near future a dam will be constructed in the river creating a fresh water reservoir. As a consequence, the length of the river section that is connected to the sea will be reduced to 65 km. All other basic parameters, including κ, will remain unchanged.



II.5[1] Explain how the amplification factor will be affected by the construction of the dam, distinguishing between the two tidal components. A computation is necessary nor sufficient.

Question III (canal connecting reservoirs)

Two large drinking water reservoirs, denoted with A and B respectively, are connected by means of a rectangular canal with a horizontal bottom and a length ℓ of 600 m. At both ends of the canal adjustable gates are situated that can be used to manipulate the flow between the reservoirs. Initially, these gates are closed. The initial depth d_0 in the canal amounts to 3.65 m. The reference level is chosen equal to the initial water level h_0 in the canal. See the figure below.



At time $t = 0$, the gate at reservoir A is opened fully and abruptly. This results in a positive translatory wave in the canal with a specific discharge q of 1.5 m^2/s. In- and outflow losses, and other effects related to the local velocity head can be neglected for this gate.

III.1[1] Use linear wave theory to compute the water level $h_{\mathrm{res,A}}$ in reservoir A.

At time $t = 0$, the gate at reservoir B is opened partially and abruptly. This results in a negative translatory wave in the canal with a height δh of -0.16 m. The partially opened gate has a head loss coefficient ξ of 25.

III.2[1.5] Use linear wave theory to compute the water level $h_{\mathrm{res,B}}$ in reservoir B.

To study the ongoing wave motion in the canal resulting from the simultaneous opening of the respective gates the method of characteristics is used.

III.3[1] Formulate the initial conditions for U and h at time $t = 0$ for $0 < s < \ell$ and the corresponding boundary conditions at $s = 0$ and $s = \ell$, respectively, for $0 < t < 200$ s.

The characteristic equations are linearized as follows

$$\frac{\mathrm{d}h}{\mathrm{d}U} = \mp\frac{c_0}{g} = \mp 0.6\,\frac{\mathrm{m}}{\mathrm{m/s}} \quad \text{for} \quad \frac{\mathrm{d}s}{\mathrm{d}t} = \pm c_0 = \pm 6\,\mathrm{m/s}$$

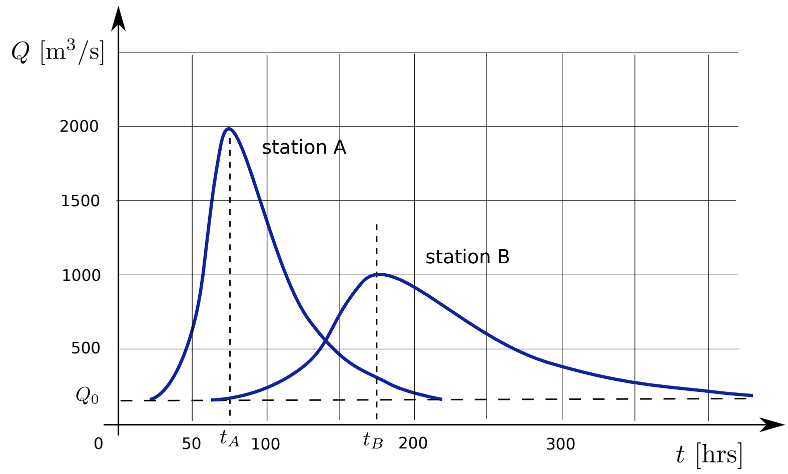
III.4[2] For $0 < s < \ell$ and $0 < t < 200$ s, plot the corresponding s,t-plane and the U,h state diagram. Provide the boundary and initial conditions with their respective numerical values and mark the successive states with Roman numbers. Use $h_{\mathrm{res,A}} =$ 0.25 m and $h_{\mathrm{res,B}} =$ -0.25 m if you were not able to compute the answers to Questions III.1 or III.2, respectively.

At time $t_1 \gg 200$ s the gate at reservoir B is closed fully and abruptly.

III.5[1.5] Compute the height of the resulting translatory wave.

Question IV (river flood wave)

From time to time, severe flooding occurs along a particular reach of a river. These floodings are caused by sudden, heavy rainfall in the catchment area of the river. For one such flooding event, the figure below shows the corresponding hydrographs (i.e. *discharge* versus time) in two stations A and B along the river, respectively, where B is situated downstream of A.



IV.1[1] Explain which feature(s) in the observed hydrographs is (are) typical of diffusive flood waves.

The river reach has a constant width $B = 3\,B_c = 700\,\mathrm{m}$, and hydraulic radius $R = d$. At time $t = t_{\mathrm{A}}$ the hydraulic radius at station A amounts to 6.5 m.

IV.2[1] Compute the high-water wave speed c_{HW} in station A at time $t = t_{\mathrm{A}}$.

The bed slope i_b and friction coefficient c_f of the considered river reach are constants equal to 7×10^{-5} and 4×10^{-3}, respectively.

IV.3[1.5] Compute the depth gradient $(\partial d/\partial s)$ at station A and time $t = t_{\mathrm{A}}$, and verify that the flood wave observed in station A at $t = t_A$ indeed has a diffusive character.

The observed $Q - h$ curve (not shown here) in station A has a more pronounced hysteresis than the $Q - h$ curve observed in station B.

IV.4[1] Explain why this is the case.

At station B the kinematic wave approximation holds.

IV.5[1.5] Compute the maximum water depth at station B during passage of the flood wave.

Answers

Question I (compression wave)

I.1 Use the wave height to discharge relation $\delta q = c\delta h$, with $c = \sqrt{g\,d_0}$ (linearization, so influence of wave height on wave speed is neglected). Using $d_0 = 0.25\,\mathrm{m}$ (giving $c = 1.57\,\mathrm{m/s}$), and using $\delta q_{\mathrm{max}} = 0.2\,\mathrm{m^2/s}$ results is a wave height δh_{max} of $0.13\,\mathrm{m}$. Linearization of the problem requires that the wave height to depth ratio $\delta h/d_0 \ll 1$ which is obviously not the case here ($\delta h/d_0 \approx 0.5$).

I.2 As stated, along positive characteristic the Riemann-invariant $R^+ = U + 2\sqrt{gd}$ is constant. All negative characteristics (in the entire domain) are issued from the undisturbed region where $U = 0$ and $c = \sqrt{gd_0}$. Hence $R^- = U - 2\sqrt{gd} = -2\sqrt{gd_0}$, everywhere. As a consequence, both R^+ and R^- are constant along positive characteristics resulting in the state (U, d) being constant along positive characteristics. (Note that U and d may vary *between* positive characteristics.)

I.3 Both U and d are constant along positive characteristics (see Question II.2). The corresponding characteristic speed $U + c = U + \sqrt{gd}$ (which is the characteristic slope $\mathrm{d}s/\mathrm{d}t$ in the s,t-plane) is therefore constant along these characteristics resulting in a straight line. These lines converge because for a compression wave the flow velocity and water depth increase with time in a fixed point of observation. Keeping s fixed, the characteristic speed $U + c$ increases as time t increases giving converging lines.

I.4 Along K_P^+ the depth d_P and flow velocity U_P are constant. In the entire domain R^- is constant leading to $U - 2\sqrt{gd} = -2\sqrt{gd_0}$, everywhere. It follows that $U_P = 2\sqrt{g\,d_P} - 2\sqrt{g\,d_0}$. Substituting $d_P = 0.30\,\mathrm{m}$ results in $U_P = 0.30\,\mathrm{m/s}$. The characteristic slope $\mathrm{d}s/\mathrm{d}t = U + c = U_P + \sqrt{g\,d_P}$ of K_P^+ is given by (substitute the previous expression for U_P) $\mathrm{d}s/\mathrm{d}t = 3\sqrt{gd_P} - 2\sqrt{gd_0} = 2.01\,\mathrm{m/s}$.

I.5 Using the wave height to discharge relation $\delta q_{\mathrm{max}} = (d_1 - d_0)\,c_{\mathrm{bore}}$ and substituting the given expression for c_{bore} leads to $\delta q_{\mathrm{max}} = (d_1 - d_0)\sqrt{g(d_1/d_0)(d_1 + d_0)/2}$. Solution for d_1 (iteration: start with $d_1 = d_0$, compute c_{bore}, compute $d_1 - d_0 \to d_1$, and repeat until convergence) gives $d_1 = 0.35\,\mathrm{m}$ and a corresponding wave height $d_1 - d_0 = 0.10\,\mathrm{m}$. This is a smaller value than the previously computed value using linear wave theory (Question I.1). The speed of a bore is larger than the speed of a gravity wave ahead of it ($c_{\mathrm{bore}} > \sqrt{g\,d_0}$) resulting in a smaller wave height when using the wave height to discharge relation for a positive translatory wave (which reads the same for linear and non-linear waves, provided the correct expression for the wave speed is used).

Question II (tidal river)

II.1 The frequency ω influences the wave length in absence of friction L_0 of progressive harmonic waves (and the corresponding wave number $k_0 = 2\pi/L_0$), and the relative resistance $\sigma = \kappa/\omega$. With increasing frequency the wave length L_0 decreases (wave number k_0 increases). An decreasing frequency increases the relative resistance. For significant values of σ (> 0.05, say) a decreasing frequency thus results in

- increased damping rate (μ) of the surface and discharge amplitudes
- reduction of the wave speed
- reduction of the wave length relative to L_0 (L_0 itself increases)

- reduction of the discharge amplitude relative to the surface amplitude
- increased phase lag (δ) between the discharge and surface elevation

II.2 The first occurrence of resonance is for $\ell/L_0 = 1/4$ (quarter wavelength resonance). Using $L_0 = c_0\,T_0 = \sqrt{gA_c/B}\,T_0$ is follows that $T_0 = 7.71 \times 10^4$ s and $\omega_0 = 2\pi/T_0 = 8.15 \times 10^{-5}$ rad/s.

II.3 The two parameters that determine the behavior of a harmonic standing wave are the basin length to wave length ratio ℓ/L_0 (expressed alternatively as $k_0\ell$), and the relative resistance $\sigma = \kappa/\omega$. For the M_2 component the basin length to wave length ($L_0 = c_0\,T_0$) ratio equals 0.43, and the relative resistance (σ) equals 0.36. The wave length is close to twice the basin length, while damping is moderately small resulting in an amplification factor around 1. For the K_1 component the wave length to basin length ratio equals 0.21 while the relative resistance of 0.71 is moderate. The situation is close to resonance (basin length is nearly a quarter wave length), although relieved somewhat by the moderate damping, causing the amplification factor to be well above 1.

II.4 The response factor is given by (formula sheet) $r = \hat{\zeta}_{\text{end}}/\hat{\zeta}_{\text{sea}} = 1/\sqrt{\cos^2 k\ell + \sinh^2 \mu\ell}$ where $k = k_0/\sqrt{1-\tan^2\delta}$, $\mu = k\tan\delta$ and $\delta = 0.5\arctan\sigma$. For the K_1 component the wave number $k_0 = 2\pi/L_0 = \omega_{K1}/c_0 = 1.0 \times 10^{-5}$ rad/m and $\delta = 0.5\arctan 0.71 = 0.31$ rad, resulting in a wave number $k = 1.06 \times 10^{-5}$ rad/m and damping parameter $\mu = 3.38 \times 10^{-6}$ 1/m. This results in an amplification factor of $r = 2.02$ and surface elevation amplitude in the closed end of $r \times \hat{\zeta}_{\text{sea,K1}} = 0.71$ m.

II.5 The shortening of the basin reduces the basin length to wave length ratio. The relative resistance remains the same, since in this example κ is not influenced (which is of course a simplification). For the M_2 component the basin length to wave length ratio becomes 0.21 which is close to quarter wave length resonance. Combined with the moderately weak damping the amplification factor increases considerably compared the the old situation (a computation reveals $r = 2.95$). The K_1 component has a basin length to wave ratio in the new situation of 0.10, which is well away from the occurrence of resonance, and consequently the amplification factor reduces significantly compared to the old situation (a computation reveals $r = 1.24$).

Question III (canal connecting reservoirs)

III.1 The translatory wave has a specific discharge $\delta q = 1.5\,\text{m}^2/\text{s}$ (given) and a wave height $\delta h = \delta q/c$. Using $c = \sqrt{g\,d_0} = 6.0$ m/s gives a wave height $\delta h = 0.25$ m. Since inflow effects related to the velocity head are neglected $h_{\text{res,A}} = h_0 + \delta h = 0.25$ m.

III.2 For the negative translatory wave the wave height to discharge relation reads $\delta q = -\sqrt{g\,d_0}\delta h = 0.96\,\text{m}^2/\text{s}$. The corresponding flow velocity equals $\delta U = \delta q/d_0 = 0.26$ m/s. This results in a head loss over the gate Δh of $\xi(\delta U)^2/2g = 0.09$ m. Since $h_{\text{res,B}} + \Delta h = h_0 + \delta h$, the reservoir level $h_{\text{res,B}}$ =(0 - 0.16 - 0.09) m= -0.25 m.

III.3 The initial condition for the wave problem reads: $U(s,0) = 0$ and $h(s,0) = 0$ for $0 < s < 600$ m At the left boundary (reservoir A, $s = 0$) the water level is imposed: $h(0,t) = h_{\text{res,A}}$ for $0 < t < 200$ s. At the right boundary (reservoir B, $s = \ell$) a discharge relation is imposed resulting in $h(\ell,t) = h_{\text{res,B}} + \xi|U(\ell,t)|\,U(\ell,t)/2g$ for $0 < t < 200$ s.

III.4 The travel time t_L of the wave in the canal equals $\ell/c_0 = 100$ s, and the considered time interval is therefore equal to two times the travel time. At time $t = 0$ two discontinuities enter the domain at the respective boundaries. See the s,t-plane and state diagram below.

III.5 For $200\,\text{s} \ll t < t_1$ the flow in the canal is steady and uniform with a water level $h = h_{\text{res,A}} = 0.25$ m and flow velocity $U = \sqrt{2g(h_{\text{res,A}} - h_{\text{res,B}})/\xi} = 0.63$ m/s (intersection point of the boundary conditions in the state diagam). Closure of the gate at reservoir B at time $t = t_1$ will then lead to a velocity variation $\delta U = -U =$ -0.63 m/s at this boundary leading to a positive translatory wave traveling in the negative direction with a height $\delta h = -d_0\delta U/c = -d_0\delta U/(-\sqrt{gd_0}) = 0.38$ m.

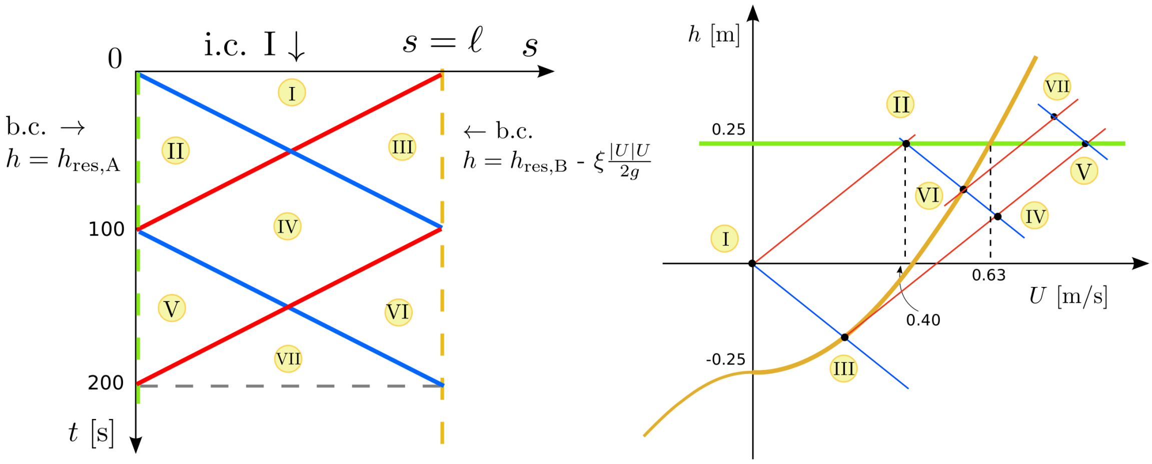


Figure 1: Question III.4: s, t-plane (left) and state diagram (right)

Question IV (river flood wave)

IV.1 Both hydrographs are asymmetrical, the discharge (and water level) rises more quickly before arrival of the wave peak than it falls after the peak has passed. This is a consequence of the flattening of the wave as time passes, caused by the different discharges (for the same depths) on the leading edge and trailing edges of the wave, respectively. This is the case if the influence of the depth gradient on the discharge can not be ignored, defining the diffusive wave type. This effect also causes a diminishing wave height (and maximum discharge) as the wave travels down the river, which is clearly visible in the observed hydrographs (station B, situated downstream of station A, having a smaller maximum discharge than station A).

IV.2 The high water wave speed is given by $c_{\text{HW}} = 1.5(B_c/B)U$, where U is the flow velocity. At time t_A, the flow velocity at station A equals the local discharge divided by the cross-sectional area, which using $R = d = 6.5\,\text{m}$ and $A_c = B_c d = 1516\,\text{m}^2$ gives $U = 1.32\,\text{m/s}$ and $c_{\text{HW}} = 0.66\,\text{m/s}$.

IV.3 Application of the quasi-steady approximation (as usual for high water waves) gives $U = \sqrt{gRi_f/c_f}$, where $i_f = -\partial h/\partial s$ is the so-called friction slope. Inserting numerical values gives $i_f = 1.1 \times 10^{-4}$. By definition $i_f = i_b - \partial d/\partial s$ resulting in a depth gradient of -3.91×10^{-5}. At station A and time t_A, the ratio of the magnitude of depth gradient and the bed slope equals approximately 0.5, and it can be concluded that the wave indeed has a diffusive character (diffusive effect can be neglected only if $\partial d/\partial s \ll i_b$).

IV.4 The hysteresis is caused by the influence of the depth gradient leading to different discharges in points having the same depths situated on the leading edge and trailing edge of the wave, respectively. This difference causes the wave to flatten as it travels down the river thereby decreasing the depth gradient and the associated hysteresis effect. At station B the wave has flattened considerably with respect to the situation in station A and the observed hysteresis in the $Q - h$ curve is therefore less pronounced (if not absent) compared to that in station A.

IV.5 In the kinematic wave approximation the friction slope is approximated by the bed slope with the discharge given by $Q = A_c\sqrt{gRi_b/c_f}$. Using $R = d$ this can be rewritten as $Q = B_c d\sqrt{gdi_b/c_f}$. Rewriting gives the water depth as a function of the discharge and cross-sectional parameters, $d^{3/2} = (Q/B_c)\sqrt{c_f/gi_b}$. The maximum discharge at station B equals $1000\,\text{m}^3/\text{s}$ leading to a maximum depth of $4.75\,\text{m}$ (in a kinematic wave the maximum discharge and maximum depth occur simultaneously).

Delft University of Technology

Faculty of Civil Engineering and Geosciences

Section of Environmental Fluid Mechanics

Exam CTB3350 / CIE3310-09

'Open Channel Flow'

Wednessday April 15th 2015, 9:00 to 12:00 hours

The number of points that can be obtained for each subquestion is shown next to the respective question number. When grading your exam, special attention will be given to a correct application of the underlying theory and a systematic approach to the problem. Provide your answers with a short description of the approach chosen. If you need to make assumptions or simplifications briefly explain why it is allowed to do so. Where a previous answer is needed in a subsequent question which you were not able to compute, assume a plausible numerical value. Numerical answers should contain the correct dimensional units, at least in the final answer but preferably also in the intermediate steps.

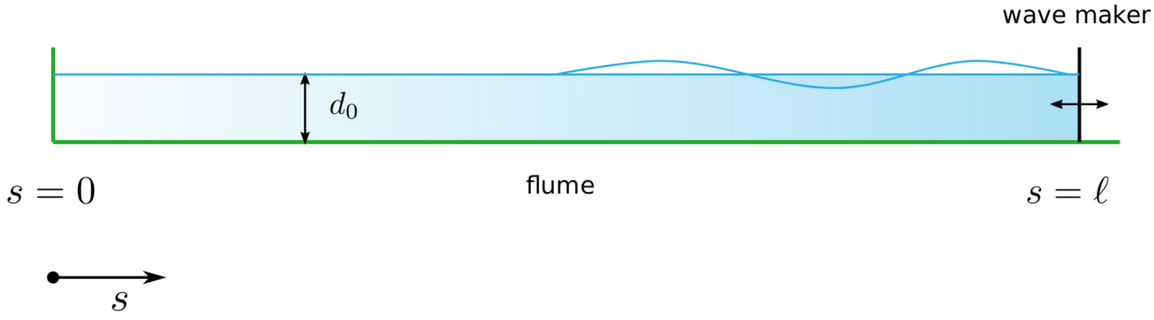
Unless explicitly stated otherwise use: mass density water $\rho_w = 1000\,\mathrm{kg/m^3}$, gravitation $g = 9.8\,\mathrm{m/s^2}$ and frequency M_2-tide $= 1.4 \times 10^{-4}\,\mathrm{rad/s}$.

Provide your answers on a separate form for each question.

Don't forget your name and student number on each form!

Question I (wave flume)

A laboratory flume with a length ℓ has a horizontal bottom and rectangular cross-section. The flume has a fixed wall at one end ($s = 0$), and a wave generator at the other end ($s = \ell$). The wave generator consists of a paddle (occupying the entire cross-section) that can move horizontally. See the figure below.



The wave generator is switched on. The horizontal, sinusoidal motion of the paddle generates periodic harmonic waves in the flume with a period T of 5 s. The undisturbed depth d_0 in the flume equals 30 cm.

I.1[1] Verify by means of a calculation that the resulting wave in the flume is a shallow-water wave.

After some time the wave in the flume becomes a periodic standing wave. The corresponding value of $\sigma = \kappa/\omega$ is sufficiently small to neglect the influence of the resistance. Furthermore, the wave height is much smaller than the water depth, allowing the use of linear wave theory.

I.2[1.5] Derive an expression relating the velocity amplitude distribution $\hat{u}(s)$ in the flume to the imposed velocity amplitude $\hat{u}(\ell)$ at the wave maker.

The length ℓ of the wave flume amounts to 20 m. The amplitude of the horizontal stroke of the wave paddle $\hat{a}$ equals 4 cm.

I.3[1] Compute the maximum amplitude of the flow velocity $\hat{u}_{\text{st}}$ of the standing wave.

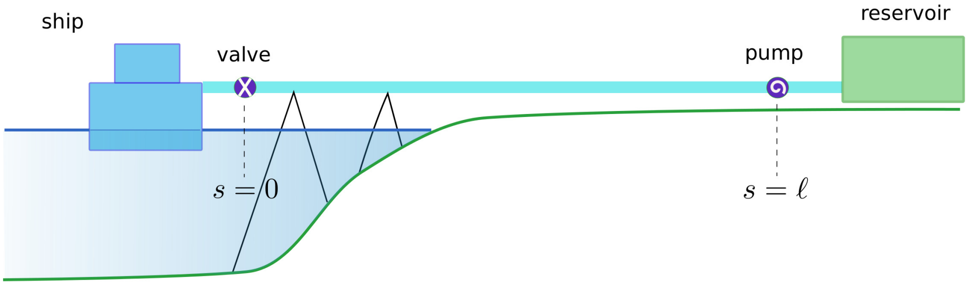
I.4[1.5] Compute the maximum surface elevation amplitude $\hat{\zeta}_{\text{st}}$ of the standing wave.

The period T of the wave generator is gradually increased until the standing wave in the flume becomes resonant. The theoretical solution for the standing wave amplitude $\hat{\zeta}_{\text{st}}$ will then tend to infinity. (Although in practice this situation is never reached.)

I.5[1] Compute the period T_{res} of this resonant standing wave and make a sketch of the corresponding wave pattern.

Question 2 (kerosine pipeline)

Kerosine is loaded from a ship into a shore based reservoir by means of a horizontal pipeline with a length ℓ. A pumping station is situated at the pipe end $s = \ell$ (reservoir) while at $s = 0$ (ship) the pipe has an emergency valve. See the schematic figure below.



The pipe has a diameter D of 1.25 m, a wall-thickness δ of 1.0 cm, and is made of steel with an elasticity modulus E of 200 GPa ($= 200 \times 10^9$ Pa). Kerosine has a bulk modulus K of 1.3 GPa and a density ρ of 800 kg/m^3.

II.1[1] Compute the speed c_0 of sound waves in kerosine and the speed c of pressure waves in the kerosine filled pipeline, and discuss whether the elasticity of the pipewall has a significant effect on the propagation of pressure waves in the pipe.

Safety regulations demand that the loading of kerosine must be stopped abruptly in case of an emergency. The shut down procedure leads to a surge wave in the pipeline whose behavior can be studied using the method of characteristics.

II.2[1] Describe *the* essence of the method of characteristics.

The initial conditions for the resulting closure problem are $U(s) = U_0$ and $h(s) = h_0$. At time $t = 0$ the pump is shut down and the valve is closed abruptly.

II.3[1.5] Formulate the boundary conditions at $s = 0$ and $s = \ell$ and make a sketch of the corresponding s, t-plane and state diagram (U, h) for the time interval $0 < t < 3t_\ell$, where t_ℓ is the travel time of pressure waves in the pipeline. Mark the initial- and boundary conditions clearly in the respective diagrams.

The pipeline has a length ℓ of 1200 m. The initial flow velocity U_0 in the pipe amounts to 1.25 m/s.

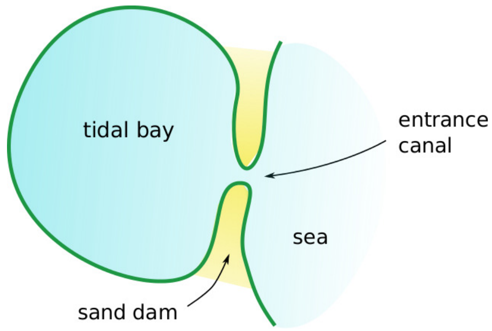
II.4[1] Compute the period T and the amplitude of the piezometric level of the resulting periodic wave in the pipeline.

In order to prevent damage to the pipeline, the increase of the tension in the pipewall ($\mathrm{d}\sigma$) due to the pressure peak should remain below 200 MPa.

II.5[1.5] Compute the maximum allowable initial velocity $U_{0,\max}$ in the pipeline. Hint: consider the balance of forces for a control volume consisting of a half cross-section of the pipe.

Question III (sand closure)

For future use as a fresh water reservoir, a tidal bay will be closed off from the sea by a dam across its entrance. During the final stages of the construction, by means of a so-called sand closure, the bay will be connected to the sea by a narrow canal only. The response of the bay to the M_2-tide at sea and the resulting currents in the temporary entrance are important issues to consider in designing the closure. See the figure below.



Subjected to tidal forcing, the nearly-closed bay and the temporary entrance behave as a small basin and a rigid column, respectively.

III.1[1] Explain the essence of the small-basin and rigid-column approximations, respectively, and mention the conditions in which these simplifications are valid.

The bay has a surface area A_b of $7\,\text{km}^2$. During the final construction phase the canal has a length ℓ of $200\,\text{m}$, a width B of $40\,\text{m}$ and a depth d of $5\,\text{m}$.

III.2[1] Determine by means of calculation whether inertia plays an important role in the resulting discrete system or not.

The bottom fricion coefficient c_f of the canal equals 3×10^{-3} (dimensionless). The tidal surface elevation at sea has an amplitude $\hat{\zeta}_{\text{sea}}$ of $1.35\,\text{m}$.

III.3[1.5] Compute the amplitudes of the surface elevation in the basin ($\hat{\zeta}_{\text{b}}$) and the discharge in the entrance ($\hat{Q}$), respectively.

The bay will be closed by a series of sand dumps in the entrance canal. In order to minimize wash-out of sand, it must be supplied when the flow velocity in the entrance equals zero. On a particular day high water at sea is at 13.35 pm.

III.4[1] Determine at which time on this day the next sand dump must be executed.

To complete the closure, the contractor has two options. Either the entrance is closed by decreasing its width or, alternatively, it is closed by decreasing its depth. The option giving the smallest flow velocities in the entrance for the same cross-section A_c is preferred (since this leads to minimal loss of sand).

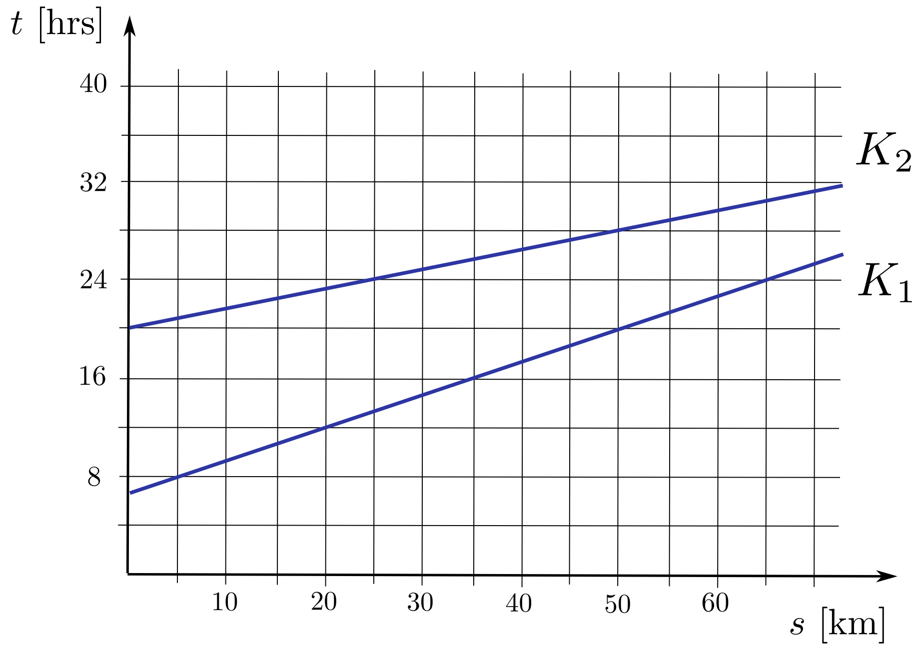
III.5[1.5] Discuss which of the two option is the preferred one. You may use a calculation in your answer, but this is considered necessary nor sufficient.

Question IV (river flood wave)

Along a particular river reach flood waves occur frequently. Flood waves in general can be modeled either as kinematic waves or as diffusive waves, depending on the situation.

IV.1[1] Explain the main differences in behavior between kinematic flood waves and diffusive flood waves, by discussing the underlying physical principles.

In order to study the behavior of flood waves along the considered river reach the kinematic wave model is applied, assuming a uniform bed slope i_b and width $B = 1.5\,B_c$ of the river. The figure below shows two characteristics K_1 and K_2, respectively, of the solution obtained with this model.



IV.2[1] Explain why the characteristics of a kinematic flood wave in a uniform channel are always straight lines.

The bed slope i_b of the river equals 2×10^{-4}, the dimensionless resistance coefficient c_f amounts to 4×10^{-3}, and the river is sufficiently wide to assume that $R \approx d$.

IV.3[1.5] Compute the depths d_1 and d_2 of the flood wave along the characteristics K_1 and K_2, respectively.

The storage width B of the river reach equals 500 m.

IV.4[1] Compute the discharge gradient $\partial Q/\partial s$ at location $s = 50$ km and time $t = 24$ hours. Hint: use the continuity equation.

IV.5[1.5] Verify whether the kinematic wave approximation is valid in this case by computing an estimate of the depth gradient during the given time interval.

Answers

Question I (wave flume)

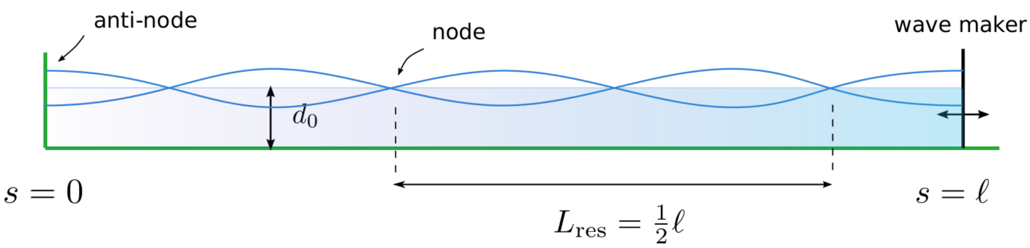
I.1 Compare the wave length to the water depth: $L_0 = c_0 T = \sqrt{g\, d_0}\, T = (1.71 \text{ m/s}) \times (5 \text{ s}) = 8.57\,\text{m}$. The wave length to water depth ratio $L_0/d_0 = 28.6$, which is larger than 20, therefore the wave is a shallow water (or 'long') wave.

I.2 The general solution for the discharge amplitude in a channel closed in $s = 0$ reads (formula sheet): $\hat{Q}(s) = Bc\,\hat{\zeta}(0)\cos\delta\,\sqrt{\sinh^2 \mu s + \sin^2 ks}$. After dividing by $A_c = Bd_0$ and for $\sigma = 0$ the velocity amplitude in the flume is given by $\hat{u}(s) = (c_0/d_0)\,\hat{\zeta}(0)\,|\sin k_0 s|$. The value for $\hat{\zeta}(0)$ follows from the boundary condition: $\hat{u}(\ell) = (c_0/d_0)\,\hat{\zeta}(0)\,|\sin k_0\ell|$, which finally results in $\hat{u}(s) = \hat{u}(\ell)\,|\sin k_0 s|/|\sin k_0\ell|$.

I.3 Using the previous result the maximum velocity amplitude of the standing wave $\hat{u}_{\text{st}}$ is given by $\hat{u}(\ell)/|\sin k_0\ell|$. The wave number $k_0 = 2\pi/L_0 = 0.733\,\text{rad/m}$, and the velocity amplitude at the wave maker is given by $\hat{u}(\ell) = \omega\hat{a} = 5.03\,\text{cm/s}$ (using $u(\ell) = \text{d}a/\text{d}t$, where a is the horizontal position of the wave maker given by for instance $a = \hat{a}\sin\omega t$ and $\omega = 2\pi/T = 1.257\,\text{rad/s}$). Substitution finally gives $\hat{u}_{\text{st}} = 5.79\,\text{cm/s}$.

I.4 The surface amplitude in the closed end (anti-node) is equal to the surface amplitude of the standing wave: $\hat{\zeta}_{\text{st}} = \hat{\zeta}(0)$. Using the intermediate result from Question I.2 it follows that $\hat{\zeta}_{\text{st}} = \hat{\zeta}(0) = (d_0/c_0)\,\hat{u}(\ell)/|\sin k_0\ell|$. Substitution of $c_0 = 1.71\,\text{m/s}$ gives $\hat{\zeta}_{\text{st}} = 1.01\,\text{cm}$.

I.5 Resonance occurs when $\sin k_0\ell \downarrow 0$, so for $k_0\ell = 0, \pi, 2\pi, \cdots$, or alternatively, $\ell/L_0 = \frac{1}{2}n$ where $n = 1, 2, \cdots, \infty$. For a wave period T of 5 s this ratio equals $\ell/L_0 = 2.33$. The first occurrence of resonance for increasing T (L_0 increases) is therefore when $\ell/L_0 = 2$. This gives a resonant wave length L_{res} of $\ell/2 = 10\,\text{m}$ and a corresponding resonant wave period T_{res} of $L_{\text{res}}/c_0 = 5.83\,\text{s}$. This resulting standing wave has an anti-node in $s = 0$ and another anti-node in $s = \ell$, see below figure.

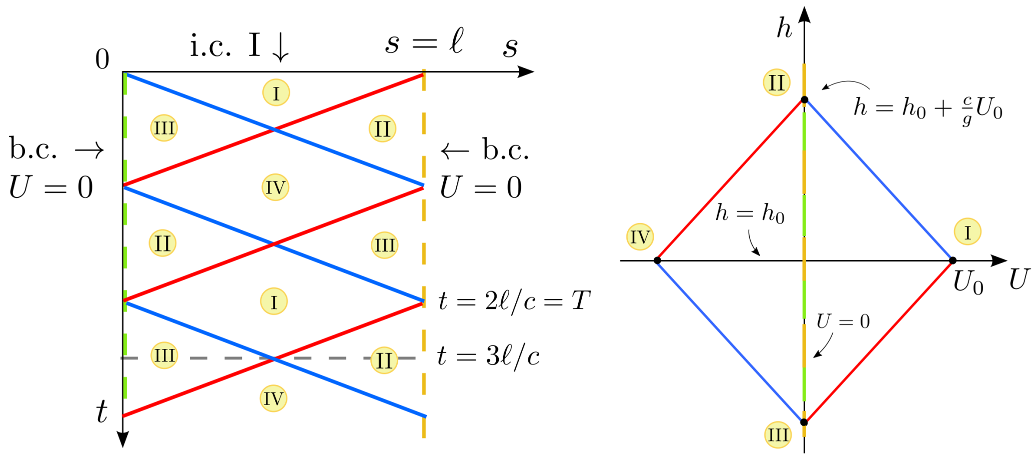


Question II (kerosine pipeline)

II.1 The sound speed c_0 in a fluid is given by $c_0 = \sqrt{K/\rho}$. For kerosine, using the given values for K and ρ this gives $c_0 = 1274.8\,\text{m/s}$. For the pipe filled with kerosine the speed of pressure waves follows from $1/c^2 = \sqrt{\rho/K + \rho D/E\delta}$ which after substitution gives $c = 946.9\,\text{m/s}$. The difference between c_0 and c is caused by elasticity of the pipewall, which reduces the wave speed (so $c_0 > c$). Comparing the respective numbers (relative decrease of ca. 30 %) it can be concluded that pipe-wall elasticity has a significant influence on the propagation of pressure waves in this case.

II.2 The two equations describing the propagation of pressure waves in pipelines form a coupled system with two unknowns, to be solved simultaneously (a so-called hyperbolic system). By transforming these equations (characteristic transformation) they can be decoupled and solved separately. The resulting two equations reduce to ordinary differential equations along particular directions in the s, t-plane, given by the characteristic slopes. In absence of forcing terms these equations state that particular variables, the Riemann invariants, are constant along the corresponding characteristics.

II.3 The boundary conditions are given by $U(0) = 0$ and $U(\ell) = 0$, respectively, for $t > 0$. The resulting s, t-plane for $0 < t < 3t_\ell$ and the corresponding state diagram are shown in the figure below.



II.4 The state in the pipe repeats after $t = 2t_\ell$, so the period of the periodic motion is given by $T = 2\ell/c = 2.0\,\text{s}$. The amplitude of the piezometric level is given by $\hat{h} = h_{\text{II}} - h_0 = h_{\text{II}} - h_{\text{I}}$. Using the characteristic relation $\mathrm{d}h/\mathrm{d}U = -c/g$ from state point I to point II (positive characteristic), and using $\mathrm{d}U = U_{\text{II}} - U_{\text{I}} = -U_0$, it follows that $\mathrm{d}h = h_{\text{II}} - h_{\text{I}} = +U_0\,c/g$ giving a piezometric level amplitude of $U_0\,c/g = 120.8\,\text{m}$.

II.5 The increase of the tension in the pipe wall due to the pressure wave follows from a balance of forces over a half cross-section: $\mathrm{d}p\,D = 2\mathrm{d}\sigma\,\delta$, or expressed in terms of the piezometric level, $\rho g \mathrm{d}h\,D = 2\mathrm{d}\sigma\delta$. Substituting the given maximum value for $\mathrm{d}\sigma$ gives a maximum increase of the piezometric level of $\mathrm{d}h_{\max} = 2\mathrm{d}\sigma_{\max}\delta/(\rho g D) = 408.1\,\text{m}$. Using the characteristic relation (see also Question II.4) this gives a corresponding $U_{0,\max} = \mathrm{d}h_{\max}\,g/c = 4.22\,\text{m/s}$.

Question III (sand closure)

III.1 Small basin approximation: accelerations and resistance in a semi-confined basin can be neglected. The (horizontal) water level in the basin can then be described a single value that depends on time only, discarding possible variations due to phase differences. Rigid column approximation: mass storage and phase differences in the conveying component of a system can be neglected allowing the discharge to be described by a single value that depends on time only neglecting spatial variations due to possible phase differences. Both approximations hold if the storing (small basin) respectively conveying (rigid column) components of the system are small relative to the wave length (factor of ca. 20).

III.2 The relative importance of the inertia term is given by ω^2/ω_0^2, where $\omega_0 = \sqrt{gA_c/\ell A_b} = 1.18$ $\times 10^{-3}$ rad/s. This yields $\omega^2/\omega_0^2 = 0.014 \ll 1$, giving a negligible influence of the inertia term in this system.

III.3 Use the explicit solution for the tidal basin with storage and resistance: the resistance parameter $\chi = 1/2 + c_f \ell/d = 0.65$, this results in $\Gamma = (8/3\pi)\,\chi\,(\omega A_b/A_c)^2\,\hat{\zeta}_s/g = 1.82$, and finally $r = \left(1/\sqrt{2}\Gamma\right)\sqrt{\sqrt{4\Gamma^2+1}-1} = 0.65$. The resulting amplitude $\hat{\zeta}_b = r\hat{\zeta}_s = 0.87\,\mathrm{m}$. The discharge amplitude follows from $\hat{Q} = A_b\omega\hat{\zeta}_b = 855.4\,\mathrm{m}^3/\mathrm{s}$.

III.4 High water in the bay occurs a phase lag $\theta = \cos r = 0.868\,\mathrm{rad}$ after high water at sea. At this instant $\mathrm{d}\zeta_\mathrm{b}/\mathrm{d}t = 0$, and therefore $Q = 0$. Expressed in minutes the phase lag equals $\theta \times (12 \times 60 + 25) = 103\,\mathrm{min}$. The sand dump therefore has to take place at 15:18 pm (half past 3, say).

III.5 The flow velocity amplitude in the entrance is given by $\hat{U} = \hat{Q}/A_c = \omega A_b r\hat{\zeta}_\mathrm{s}/A_c$. For a given cross-sectional area A_c this value is smallest for the smallest possible value of r. Since r decreases with increasing Γ, and Γ is proportional to χ (for a given A_c) the option giving the largest value for χ is the preferred one. Since $\chi = 1/2 + c_f\ell/R$ and R decreases more rapidly with d than with B (as long as $B \gg d$) the depth of the entrance channel should be decreased during the final stages of the closere rather than its width.

Question IV (river flood wave)

IV.1 Kinematic wave: the discharge in a particular cross-section depends on the cross-sectional parameters and the bed slope i_b only. In an idealized situation, two cross sections having the same depths also have the same discharges. Applied to two cross-sections having the same depths on the leading and training edges of a flood wave, respectively, it follows that a kinematic wave preserves its height, although it can deform (due to the wave speed being dependent on the water depth). Diffusive wave: the discharge is also dependent on the depth gradient. Two cross-sections on the leading and trailing edges of a flood wave with the same depths, have different discharges (due to the depth gradient effect) leading to flattening and peak height reduction of the flood wave, and a so-called hysteresis in the $Q-h$ curve.

IV.2 For a kinematic wave in a channel with uniform cross-section, the discharge, cross-sectional area, and high-water wave speed (c_HW) are functions of the local depth d. The kinematic wave solution states that the depths are constant along characteristics $\mathrm{d}s/\mathrm{d}t = c_\mathrm{HW}$. Since the depth determines the value of c_HW, it follows that the characteristics have a constant slope and are thus straight lines.

IV.3 The characteristic speeds $c_{\mathrm{HW},1}$ and $c_{\mathrm{HW},2}$ along the respective characteristics K_1 and K_2 can be determined from the slopes of these lines. This gives $c_{\mathrm{HW},1}$ = (15 km) / (4 hrs)= 1.04 m/s, and $c_{\mathrm{HW},2}$ = (25 km) / (4 hrs)= 1.74 m/s. Using $B_c/B = 2/3$ it follows that the associated flow velocities $U_1 = c_{\mathrm{HW},1} = 1.04\,\mathrm{m/s}$ and $U_2 = c_{\mathrm{HW},2} = 1.74\,\mathrm{m/s}$. Finally from $U = \sqrt{gd\,i_b/c_f}$ the depths can be found: $d_1 = 2.21\,\mathrm{m}$ (depth along K_1), and $d_2 = 6.15\,\mathrm{m}$ (depth along K_2).

IV.4 From the continuity equation $\partial Q/\partial s = -B\partial d/\partial t$. At $s = 50\,\mathrm{km}$ and for $t \approx 24\,\mathrm{hrs}$ the depth changes from d_1 to d_2 during a time interval Δt of 8 hrs (*vertical* distance between the two characteristics). Using $d_2 - d_1 = \Delta d$, the right hand side of the continuity equation above can now be estimated as $-B\Delta d/\Delta t \approx$ -(500 m) $\times$ (3.94 m)/(8 $\times$ 3600 s)= -6.8 $\times 10^{-2}\,\mathrm{m}^3\mathrm{s}^{-1}/\mathrm{m}$. Note the minus sign, the discharge decreases in positive s-direction (leading edge of the flood wave).

IV.5 The kinematic wave approximation holds if the depth gradient $\partial d/\partial s$ is very small relative to the bed slope i_b (their ratio being $\ll 1$). The depth gradient can be estimated from the horizontal distance between the two characteristics. For instance at time $t = 24\,\mathrm{hrs}$ this distance Δs equals 40 km. The magnitude of the depth gradient is therefore approximately Δd / (40,000 m)$\approx 10^{-4}$ which is about half the bed slope. The restriction on the depth gradient allowing the use of the kinematic wave approach is therefore (by far) not met. Instead, the diffusive wave approach should have been used.

Delft University of Technology

Faculty of Civil Engineering and Geosciences

Section of Environmental Fluid Mechanics

Exam CTB3350 / CIE3310-09

'Open Channel Flow'

Monday April 14th 2014, 18:30 to 21:30 hours

The number of points that can be obtained for each subquestion is shown next to the respective question number. When grading your exam, special attention will be given to a correct application of the underlying theory and a systematic approach to the problem. Provide your answers with a short description of the approach chosen. If you need to make assumptions or simplifications briefly explain why it is allowed to do so. Where a previous answer is needed in a subsequent question which you were not able to compute, assume a plausible numerical value. Numerical answers should contain the correct dimensional units, at least in the final answer but preferably also in the intermediate steps.

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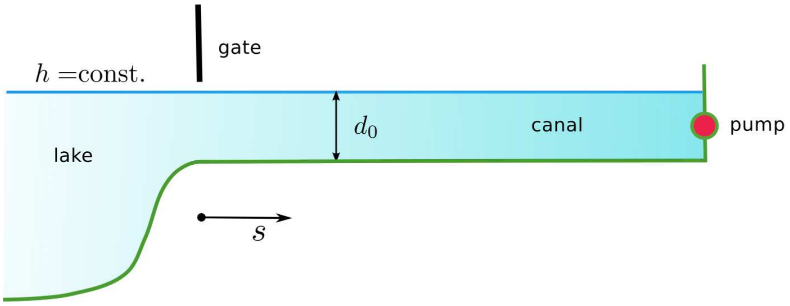
Provide your answers on a separate form for each question.

Don't forget your name and student number on each form!

Question I (intake canal)

An intake canal is connected to a large lake (with constant water level) by means of an adjustable gate. At the other end of the canal a pumping station is situated. Initially, the gate is fully opened with a zero discharge in the canal. See the figure below.



In this question the wave motion in the canal resulting from manipulations with the pump and/or gate will be studied using the elementary wave equation.

I.1[1] The reduction of the general one-dimensional shallow water equations to the elementary wave equation comprises several steps. Mention these steps and discuss when these are allowed.

The canal has a horizontal bottom, a constant width $B = B_s = 40\,\text{m}$ and an undisturbed depth $d_0 = 5\,\text{m}$. At time $t = 0$ the pump is switched on abruptly with a discharge (withdrawal) Q of $80\,\text{m}^3/\text{s}$.

I.2[1] Compute the speed and height of the resulting translatory wave issued in the canal. Verify that this wave can indeed be considered a 'linear' wave.

After some time the translatory wave reaches the open entrance (at time $t = t_L$) where it is reflected fully and negatively.

I.3[1] Which boundary condition holds in case of fully negative reflection? For the above case compute the wave height and and discharge in the entrance shortly after reflection.

Consider a similar situation where the gate in the entrance is partly open, giving the local boundary condition $d = d_0 - \xi|U|U/(2g)$ where $\xi = 20$ is the gate's discharge coefficient. The boundary condition at the pumping station and the initial conditions remain unchanged.

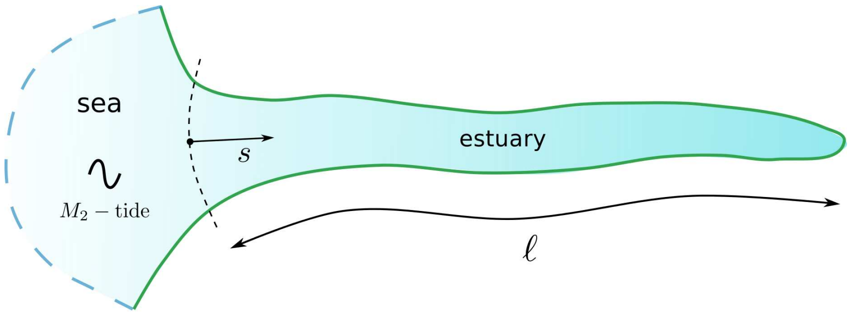
I.4[2] Construct the s,t-plane and the U,d-state diagram for $0 < t < 5t_L$ using the above initial conditions and boundary conditions and the linearised characteristic equations: $\delta d/\delta U = \mp d_0/c_0 \approx \mp\, 0.75\,\text{m}/(\text{m/s})$ for $\text{d}s/\text{d}t = \pm c_0$.

I.5[1.5] From the graphical solution, determine/compute the water depth and flow velocity at the entrance of the canal shortly after the first reflection at time $t = t_L$. Compare the result to that of question I.3 and explain the differences.

I.6[1] Describe qualitatively how the wave motion in the canal evolves for $t > 5t_L$ and compute the flow state in the canal for $t \to \infty$.

Question II (tidal wave)

An estuary with a length ℓ is openly connected to an ocean in which an M_2-tide prevails, causing a damped *progressive* wave in the estuary. See the figure below.



II.1[1] Give a possible explanation for the fact that the tidal wave in this estuary is a progressive wave (instead of a standing wave), despite the estuary being closed in one end.

The wave motion in the estuary will be studied using the harmonic method. To this end the estuary is schematised as a horizontal channel with a constant width $B = 3B_s = 6000\,\mathrm{m}$, constant depth $d = 12\,\mathrm{m}$ and linear friction factor $\kappa = 7 \times 10^{-4}\,\mathrm{s}^{-1}$.

II.2[1] Compute the wave number k, the damping parameter μ and the speed c of the (harmonic) tidal wave. Give an indication of the importance of the resistance relative to the inertia in this wave.

The complex amplitude of the surface elevation at sea $\tilde{\zeta}_{sea}$ equals $(1.75\,\mathrm{m}) \times \exp(i\pi/2)$.

II.3[1.5] Construct the hodograph of the surface elevation in the estuary. To this end, compute the amplitude and phase of the surface elevation in a sequence of points at distances of 25 km, 50 km, 75 km and 100 km, respectively, from the mouth of the estuary.

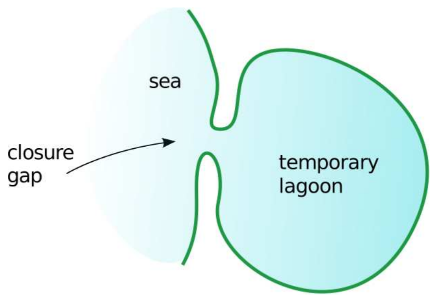
II.4[1.5] Compute the amplitude and phase of the discharge in the entrance and, following a similar procedure as in the previous question, construct the hodograph of the discharge in the estuary. Use a separate figure.

Consider now the hypothetical case where the estuary has an infinite length and the tidal wave propagating into the estuary is undamped (that is, $\kappa = 0$). The amplitude and phase of the surface elevation in the entrance are the same as in the previous (damped) case, as are the cross-sectional dimensions.

II.5[1] Describe in which respect the hodographs for the surface elevation and the discharge, in an undamped wave, respectively, would differ from the previous case of a damped progressive wave.

Question III (closure gap)

During the final phase of a land reclamation project a temporary tidal lagoon, connected to the sea by a narrow gap, is closed by dumping a large amount of sand instantaneously into the gap. To optimize the execution of this closure the contractor needs to know the hydraulic conditions in the lagoon-gap system. See the figure below.



The configuration consisting of the lagoon and gap is modeled as a discrete system with storage and resistance.

III.1[1] Which are the essential features of such a discrete system and when is this approach applicable?

For preliminary computations the discrete system is linearised. The lagoon has a surface area A_b of $2\,\mathrm{km}^2$ and the gap has a conveying cross-section A_c of $50\,\mathrm{m}^2$. The M_2-tide at sea is sinusoidal with a surface elevation amplitude of 1.25 m.

III.2[1.5] Compute the amplitudes of the surface elevation inside the lagoon and the discharge in the gap, respectively.

III.3[1] Interpret the result of question III.2 in terms of the time scale τ of the system and the frequency of the tidal forcing.

The final sand dump should take place in the afternoon as soon as the flow velocity in the gap becomes zero. On closure day high water at sea is at 1:30 pm.

III.4[1.5] Determine the preferred time instant for the sand dump.

Do to equipment failure the closure was only accomplished partially, the remaining gap cross-section being $15\,\mathrm{m}^2$ after closure. This situation may not last too long since the increased flow velocities in the gap will wash away the freshly dumped sand.

III.5[1] Explain why the flow velocities in the gap have increased compared to the situation before the (partial) closure. A computation to support your answer is not necessary nor sufficient.

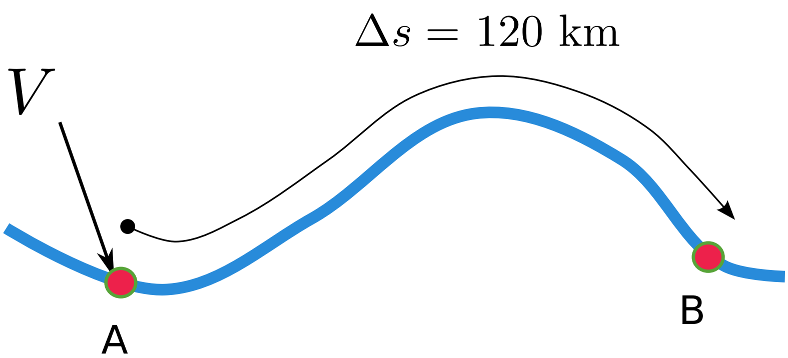
Question IV (river flood wave)

Following heavy rainfall in the upper catchment area of a river, a high-water wave is propagating in downstream direction. Initially, this wave has the character of a diffusive wave but later on its behaviour tends to that of a kinematic wave.

IV.1[1] What is the principal difference in behaviour between a diffusive high-water wave and a kinematic high-water wave?

IV.2[1] Explain why a high-water wave will gradually loose its diffusive character and ultimately tend to a kinematic wave (provided the river is long enough).

For preliminary analyses a linearised model for high water waves is used. The river is schematised using a constant cross-section, having a width $B = 2B_s = 500\,\mathrm{m}$ and depth $d = 5\,\mathrm{m}$, a bed-slope $i_b = 4 \times 10^{-5}$ and resistance coefficient $c_f = 0.003$. Rainfall is schematised as a sudden release of water into the river at point A. See the figure below.



IV.3[1] Compute the travel time of the top of the high-water wave to a point B which is situated at a distance $\Delta s = 120\,\mathrm{km}$ downstream of A.

The instantaneous release of water into the river at A, modeling the rainfall, has a volume V of $50 \times 10^6\,\mathrm{m}^3$.

IV.4[1.5] Compute the maximum water depth in B during passage of the high-water wave.

IV.5[1.5] Verify, by means of a numerical estimate, whether the high-water wave still has a diffusive character or not by the time the top of the wave passes B. (Hint: estimate the width of wave and the resulting depth gradient.)

Answers

Question I (intake canal)

I.1[1] The reduction of the general one-dimensional shallow water equations to the (linear) elementary wave equation comprises the following steps:

- The advective acceleration term is discarded, this is a good approximation if the Froude-number is small ($Fr \ll 1$), which is the case if the wave height is small with respect to the water depth ($\delta h \ll d_0$).
- The resistance term is neglected, which is allowed if the time scale of the wave problem is sufficiently small (resistance is then over ruled by inertia).
- The parameters characterising the cross-section (width B, conveyance area A_c, bottom level z_b) are constant in space and constant in time, i.e. the canal is prismatic and water level changes resulting from the wave motion hardly influence its cross-section (which is generally a good approximation if the wave height is small relative to the water depth).

(Differentiating the (linearized) continuity equation with respect to time and the momentum equation with respect to s and elimination of the discharge variable Q gives then the elementary wave equation for h or, alternatively, d.)

I.2[1] For $B = B_c$ (rectangular cross-section), the wave speed $c = \sqrt{gd_0} = 7.0\,\mathrm{m/s}$. Wave height (using wave height discharge relation for wave propagating in the negative s-direction) is equal to: $\delta h = -\delta Q/(Bc)$ =-0.30 m (note the minus sign). The elementary wave approximation holds since:

- The Froude-number $Fr = U/c = \delta h/d_0 \approx 0.06 \ll 1$.
- Uniform depth and width while $\delta h \ll d_0$ implies constant cross-sectional parameters, both in time and in space.
- Abrupt change (translatory wave), small time scale, resistance is small relative to inertia.

I.3[1] Fully negative reflection occurs when the water level at the boundary is constant, the local surface elevation due to the wave motion is then equal to zero. Therefore, shortly after reflection the wave height at the boundary equals zero ($\delta h = 0$). The corresponding discharge at the open boundary is twice that of the incoming wave, therefore $\delta Q = 160\,\mathrm{m^3/s}$, after reflection. (The latter follows from substituting the boundary condition $\delta h = \delta h^+ + \delta h^- = 0$ into the wave height - discharge relation $\delta Q = Bc\,(\delta h^+ - \delta h^-) = Bc \times 2\delta h^+$.)

I.4[2] See Figure 1 below.

I.5[1.5] The flow state at the entrance of the canal shortly after the first occurence of reflection corresponds to point III in the state diagram. The corresponding values for U and d can be determined directly from the state diagram (provided it is plotted sufficiently accurate), or they can be computed from $d = 5\,\mathrm{m} - 20U^2/2g = 4.70\,\mathrm{m} + 0.75 \times U$. The result is $U_{III} = 0.48\,\mathrm{m/s}$ and $d_{III} = 4.76\,\mathrm{m}$.

I.6[1] Initially, when the pump is switched on, a negative translatory wave is issued in the canal propagating in negative direction (from right to left). This wave is partially reflected at the left boundary

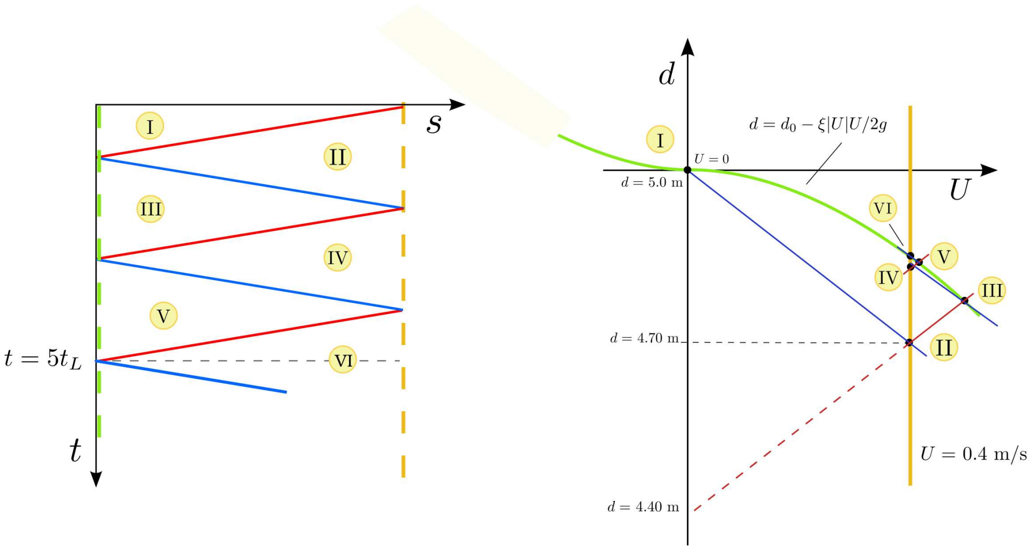


Figure 1: s, t-plane (left panel) and U, d-state diagram (right panel)

(gate) after which it propagates back towards the pumping station where it is reflected fully and positively. This sequence is repeated where each time the wave reflects from the left boundary it looses part of its height (energy) which ultimately leads to a steady flow state in the canal that satisfies both boundary consitions. Hence, for $t \to \infty$ the corresponding flow state in the canal is given by $U_\infty = Q/(B_c d_0)$ = 0.4 m/s, inserting this into the left boundary condition gives $d_\infty = d - 20 \times (0.4)^2/(2g) = 4.84$ m (or δh =-0.16 m).

Question II (tidal wave)

II.1[1] In principle, wave reflection will occur at the closed end of a basin (estuary), giving a standing wave. However, if the damping of the incoming wave is very large, the resulting wave height is nearly zero at the closed end of the basin and the reflected wave will be hardly noticable, leading to an virtually progressive wave in the estuary. This is the case for $\mu\ell$ large (¿ ca. 2, say), which is apparently the situation here.

II.2[1] First compute the wave number in absence of resistance: $k_0 = \omega/c_0$, using $c_0 = \sqrt{g \times 3B_c \times d/B} =$ 6.26 m/s, $k_0 = 2.23 \times 10^{-5}$ rad/m. The wave number $k = k_0/\sqrt{1 - \tan^2\delta}$, in which $\delta = \frac{1}{2}\arctan\sigma$. Using $\sigma = \kappa/\omega = 5$ (dimensionless) gives $\delta = 0.69$ rad (=39.35 °), and finally $k = 3.90 \times 10^{-5}$ rad/m. Next, the damping parameter $\mu = k\tan\delta = 3.20 \times 10^{-5}$ 1/m. The wave speed $c = \omega/k = k_0\sqrt{1 - \tan^2\delta} =$ 3.59 m/s.

II.3[1.5] For a singular harmonic wave propagating in the positive direction, the complex amplitude of the surface elevation is given by $\tilde{\zeta} = \tilde{\zeta}^+ = \tilde{\zeta}_0 \exp(-ps)$, in which $p = \mu + ik$. This gives for the amplitude and phase of the surface elevation, respectively, $|\tilde{\zeta}| = |\tilde{\zeta}_0| \exp(-\mu s) = (1.75\,\mathrm{m}) \times \exp(-\mu s)$ and $\arg\tilde{\zeta} = \arg\tilde{\zeta}_0 + \arg\{\exp(-iks)\} = \pi/2 - ks$. That is, starting from a phase of $\pi/2$ in $s = 0$, the phase decreases with $k \times 25000$ m =0.98 rad (55.9 °) every 25 km. The amplitude decreases from 1.75 m at $s = 0$ by a factor of $\exp(-\mu \times 25000\,\mathrm{m}) = 0.45$ each 25 km. The results are summarized in Table 1 below.

s [km]	phase ζ	$\hat{\zeta}$ [m]
0	90°	1.75
25	34°	0.79
50	-22°	0.35
75	-78°	0.16
100	-134°	0.07

Table 1: Phase and amplitude of the surface elevation for various values of s

Plotting the amplitude and phase in a polar diagram and connecting the points by a smooth curve gives the hodograph of the surface elevation, see Figure 2.

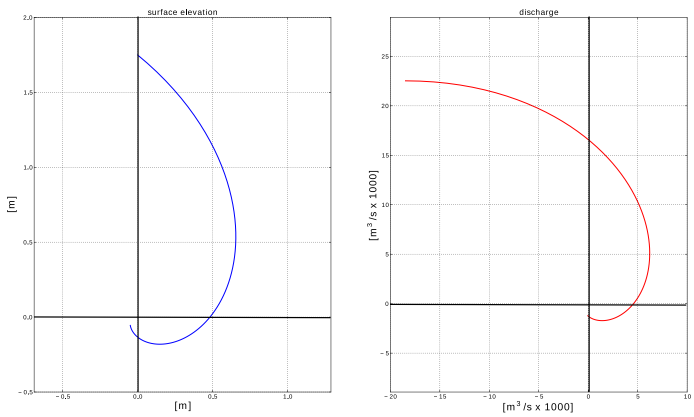


Figure 2: Hodograph of the surface elevation (left panel) and of the discharge (right panel)

II.4[1.5] For a progressive harmonic wave the complex amplitude of the discharge is given by $\tilde{Q} = Bc\cos\delta\,\exp(i\delta)\,\tilde{\zeta}$. The wave number and damping parameter are the same as for the surface elevation, i.e. the phase change and amplitude reduction factor over each 25 km are the same as for the water level. The amplitude and phase of the discharge at $s = 0$ are computed from $\hat{Q}_0 = Bc\cos\delta\hat{\zeta}_0 = 29128\,\mathrm{m}^3/\mathrm{s}$, and $\arg\tilde{Q}_0 = \arg\tilde{\zeta}_0 + \delta = 2.26\,\mathrm{rad}$ (=129°), respectively. (The hodograph for the dicharge is rotated over an angle $\delta = 39.3°$, counter clockwise, relative to the hodograph for the surface elevation.) See Table 2 and Figure 2.

s [km]	phase Q	$\hat{Q}$ [m^3/s]
0	129°	29128
25	73°	13089
50	17°	5882
75	-38°	2643
100	-94°	1188

Table 2: Phase and amplitude of the discharge for various values of s

II.5[1] In case of an undamped wave, the damping parameter $\mu = 0$ and the amplitudes of the surface elevation and discharge, respectively, do not change in the propagation direction of the wave. The respective hodographs are then circle segments instead of logarithmic spirals. Also, since the friction angle δ would be zero, the surface elevation and discharge would have the same phase everywhere (equal to $\pi/2$ in $s = 0$). Since in absence of resistance the wave number is smaller, the respective phases in a particular point would be smaller than in the previous case of a damped wave. Finally, in absence of damping, the ratio of the discharge amplitude to the surface elevation amplitude is larger.

Question III (closure gap)

III.1[1] In a discrete system with storage and resistance two functional components can be distinguished, a water storing component (the lagoon in this case) and a conveying component (the closure gap) where only resistance is important (negleting inertia). The latter will be the case if the length of the conveying component is neglegeable (more precisely, the resulting eigenfrequency is large relative to the tidal frequency). Furthermore, the wave length exceeds by far the horizontal dimensions of both components (by a factor of 20, say). In that case the water level and discharge in the storing and conveying parts can be described by single parameters, ζ_b and Q, respectively, which both depend on time only.

III.2[1.5] The response of the lagoon depends entirely on the resistance parameter $\Gamma = \frac{8}{3\pi}\chi\,(A_b\omega/A_c)^2\,\hat{\zeta}_s/g$ (inertia plays no role here). For a small cap $\chi = 1/2$ with which Γ =1.70. The response factor (amplitude ratio) is given by $r = \frac{1}{\sqrt{2}}\frac{1}{\Gamma}\sqrt{-1+\sqrt{1+4\Gamma^2}} = 0.66$, giving a surface amplitude in the lagoon of $\hat{\zeta}_b = r\hat{\zeta}_s = 0.83\,\mathrm{m}$. The corresponding discharge amplitude in the gap is given by $\hat{Q} = \omega A_b\hat{\zeta}_b = 232\,\mathrm{m}^3/\mathrm{s}$.

III.3[1] The responce to a (tidal) forcing of a discrete system with storage and resistance only is determined entirely by $\omega\tau$, where ω is the frequency of the forcing and τ is the time scale of the system (relaxation time). For $\omega\tau \ll 1$ the effect of resistance is hardly noticable and the response factor r will be nearly 1 and for $\omega\tau \gg 1$ the response of the system will be very small ($r \downarrow 0$). In this case $\omega\tau = \Gamma r = 1.13$ implying that resistance is important in this system, yet not dominant, as evidenced by the moderate value of r.

III.4[1.5] The flow velocity and discharge in the gap are zero if the water level in the lagoon reaches a maximum or minimum. This is determined by the phase lag θ (high water inside the lagoon occurs a phase angle θ after high water at sea). The phase lag can be computed from $\cos\theta = r$ giving $\theta = 0.84\,\mathrm{rad}$. (You can also use the formula on the formula sheet with $\omega/\omega_0 = 0$). The corresponding time lag $\Delta t = (\theta/2\pi)\times T$, where T is the wave period, equals 1 h 40 min. The sand dump should therefore take place at 3:10 p.m.

III.5[1] With resistance only, the maximum flow velocity in the entrance occurs when the water level difference between lagoon and sea is maximum. With a smaller cross-section of the gap, the amplitude ratio r decreases. (This can be seen directly from the expressions for Γ and r, or from the following reasoning: suppose ζ_b does not change $\rightarrow$ Q remains the same $\rightarrow$ higher velocities in gap if A_c smaller $\rightarrow$ resistance in gap increases $\rightarrow$ ζ_b decreases.) A smaller amplitude ratio gives a larger maximum difference between ζ_s and ζ_b leading to increased maximum flow velocities in the gap dusing the tidal cycle.

Question IV (river flood wave)

IV.1[1] In a kinematic high-water wave the local discharge depends on the local depth and bed slope only while in a diffusive high water wave de spatial gradient of the water depth influences the dicharge too. As a consequence of the latter, the discharge at the leading edge of a high water wave is larger than the discharge in a cross-section on the trailing edge having the same depth. This causes the wave to flatten and spread out in the course of time. A kinematic wave does not show this behavior, since points having the same depth have the same discharge. The wave may deform as it propagates down the river but keeps the same height.

IV.2[1] The flattening of the high-water wave is caused by the depth gradient in the wave and depends on the magnitude of the depth gradient relative to the bed slope. If the wave flattens, the depth gradient decreases which in turn reduces this effect. Therefore the diffusive character of the wave is gradually lost and will ultimately disappear, tending to a kinematic wave.

IV.3[1] The travel time from A to B is equal to the distance traveled by the wave divided by the wave speed: $\Delta t_{AB} = \Delta s_{AB}/c_{HW}$, where $c_{HW} = \frac{3}{2}\frac{B_c}{B}U_e$. The uniform flow velocity $U_e = \sqrt{gRi_b/c_f} = 0.81\,\mathrm{m/s}$ and $c_{HW} = 0.61\,\mathrm{m/s}$. The travel time is then $\Delta t_{AB} = 197848\,\mathrm{s}$=54 h 57 min.

IV.4[1.5] The water depth of the wave top at point B is given by $d_{top,B} = d_0 + \frac{V/B}{\sqrt{2\pi}\sigma_B}$ in which the standard deviation $\sigma_B = \sqrt{2K\Delta t_{AB}}$. The diffusion coefficient $K = Q_e/(2i_bB) = 25272\,\mathrm{m}^2/\mathrm{s}$ giving $\sigma_B = 100\,\mathrm{km}$ and finally a water depth $d_{top,B} = d_0 + 0.40\mathrm{m} = 5.40\,\mathrm{m}$.

IV.5[1.5] The discharge in a high-water wave is given by $Q = Q_e\sqrt{1 - i_b^{-1}\partial d/\partial s}$. A kinematic wave results if the depth gradient can be neglected, that is $\partial d/\partial s \ll i_b$. The total width of the high water wave by the time it passes B is approximately $4\sigma_B$ = ca. 400 km. An order of magnitude estimate for the depth gradient is then $\partial d/\partial s \approx \Delta d_{top,B}/2\sigma_B$ = ca. 2×10^{-6}. (You may also compute the maximum depth gradient by differentiating the solution for d with respect to s, this gives however the same order of magnitude estimate.) The gradient to bed slope ratio ($\partial d/\partial s/i_b$) is then approximately 0.05 which is sufficiently small to neglect the influence of the depth gradient altogether.